Curso de Doctorado

UNIVERSIDAD COMPLUTENSE MADRID

Topics in geometric analysis: Geometry of affine sphere

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Lugar: Facultad de Ciencias Matemáticas, sala 225, UCM
Horario: Viernes, 10:00-12:00. 7 octubre - 16 diciembre 2016
Número de horas: 20

Resumen: Given a group of motions, one can study the properties of hypersurfaces that are invariant with respect to this group. For the group of Euclidean motions, this leads to the classical differential geometry of hypersurfaces. Attention has historically focused on interesting classes of hypersurfaces, for example umbilic, minimal, and constant mean curvature hypersurfaces. For the group of equiaffine (volume-preserving affine) transformations, the same considerations lead to what is known as the affine differential geometry of hypersurfaces, although even the most basic objects, such as the affine normal, require more work to define than do their Euclidean counterparts. The course will start giving various definitions of the equiaffinely invariant normal vector field associated with a nondegenerate hypersurface and proving the equivalence of these notions. Once the affine normal is defined, the basic structures to be used in the study of the affine geometry of hypersurfaces can be defined: the equiaffine (Blaschke) metric, induced projective structures, and cubic (Pick) form. The simplest interesting classes of affinely invariant hypersurfaces are the affine spheres and the equiaffine mean curvature zero hypersurfaces, that are, respectively, the affine analogues of umbilic and minimal hypersurfaces in Euclidean geometry. In contrast with the Euclidean setting, inequivalent affine spheres abound and the study of affine mean curvature zero hypersurfaces is little developed. Their construction and characterization requires the study of certain differential equations of Monge-Ampere type, that relate the Hessian determinant of an unknown function with some function of the unknown function. The relevant equations will be deduced, and what is known about their solvability will be surveyed. The following topics will be surveyed, to the extent that time permits:

1. Characterization of elliptic and parabolic convex affine spheres.
2. The theorem of Cheng and Yau showing that the interior of a pointed convex cone is foliated in a unique way by hyperbolic affine spheres asymptotic to the cone.
3. Recent analogous results of Klartag regarding incomplete elliptic affine spheres.
4. Construction of homogeneous affine spheres.
5. Construction of equiaffine mean curvature zero hypersurfaces.