Colloquium del Departamento

de Análisis Matemático

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“Tingley's problem for compact operators”

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Abstract: In 1932, S. Mazur and S. Ulam solved a question posed by S. Banach by showing that every surjective isometry between real normed spaces is affine. The result is nowadays called Mazur-Ulam theorem. Many results rely on this contribution, and many consequences have been derived from it. P. Mankiewicz establishes in 1972 a generalization of the Mazur-Ulam theorem by proving that every isometry T of an open connected subset V of a normed linear space X onto an open subset W of a normed linear space Y has a unique isometric extension $\tilde T$ of X onto Y such that $\tilde T (tx+(t)y)=t \tilde T (x)+ (1-t)\tilde T (y)$ for any real number t and $x,y\in X$.

A variant of the Mazur-Ulam theorem for subsets with empty interior was considered by Tingley in 1987. Let X and Y be normed spaces, whose unit spheres are denoted by $S(X)$ and $S(Y)$, respectively. Suppose $f:S(X)\to S(Y)$ is a surjective isometry. Tingley proved that a surjective linear isometry f between the unit spheres of two finite dimensional Banach spaces preserves antipodes, that is, $f(-x) = -f(x)$, for every x in $S(X)$. The so-called \emph{Tingley's problem} asks wether a surjective linear isometry $f:S(X)\to S(Y)$ can be extended to a real-linear (bijective) isometry $T : X \to Y$ between the corresponding spaces.

Although, Tingley's problem remains open up today, several positive answers have been obtained for concrete Banach spaces. Affirmative answers to Tingley's problem are known for $\ell_p (\Gamma)$ (G.G. Ding, 2003-2015), L^p-spaces (D. Tan, 2011-2013), $C(X)$ spaces (G.G. Ding, 2003, D. Tan, X. Huang and R. Liu, 2013), finite dimensional polyhedral Banach spaces (V. Kadets and M. Mart{\'i}n, 2012), and finite dimensional C*-alegbras (R. Tanaka, 2016).
We shall present a new positive answer to Tingley's problem in the case of a surjective isometry between the unit spheres of two spaces of compact operators on arbitrary Hilbert spaces. The novelties include a new point of view provided by techniques, developed in setting of C*-algebras and JB*-triples, describing the facial structure of the closed unit ball of these structures.

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