

# Seminario de Geometría y Topología



## **New developments and applications of the Inverse Problem of the Calculus of Variations**

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### **Resumen:**

In brief, the inverse problem of the calculus of variations addresses the question of whether or not a system of second order ordinary differential equations (SODE for short) is equivalent to a regular Lagrangian system. This problem dates back to the end of the 19th century, at which time only the one-dimensional case was understood. Forty years latter, the Fields medalist J. Douglas gave a classification for two-dimensional systems. After that no other dimension has been completely classified.

Geometric mechanics refers to a variety of topics that lie at the intersection of differential geometry, dynamical systems, both discrete and continuous, and analytical mechanics. The inverse problem is the leading thread of this talk, but it runs through some central issues in geometric mechanics, namely nonholonomic systems and the Hamiltonization problem, Lagrangian mechanics on Lie algebroids, stabilization of mechanical systems using appropriate controls and discrete mechanics, in particular geometric integrators.

The main contributions are the following. We give a new geometric characterization of the classical inverse problem in terms of Lagrangian submanifolds which allows us to extend the problem to a variety of contexts, concretely to constrained mechanics, including nonholonomic mechanics, Lagrangian mechanics on Lie algebroids and discrete mechanics. Each of these extensions can be related to other problems, namely Hamiltonization of nonholonomic systems, reduction by symmetries and geometric integration. We also provide some applications of the inverse problem to control theory, more preciey to the problem of stabilization of an unstable equilibrium. Finally we introduce energy-preserving integrators for nonholonomic systems, which are not variational in the usual sense.

**Lugar: Universidad Complutense de Madrid**  
**Facultad de Ciencias Matemáticas**  
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**Fecha y Hora: Martes, 17 de enero de 2017, 12:00**

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