Colloquium del Departamento

de Análisis Matemático

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“Two conjectures of Astala on distortion of sets under quasiconformal maps and related removability problems”

Martes 20 de junio de 2017
a las 13:00 horas en el seminario 222

Abstract:

Quasiconformal maps are a certain generalization of analytic maps that have nice distortion properties. They appear in elasticity, inverse problems, geometry (e.g. Mostow's rigidity theorem)... among other places.In his celebrated paper on area distortion under planar quasiconformal mappings, Astala proved that if E is a compact set of Hausdorff dimension d and f is K-quasiconformal, then fE has Hausdorff dimension at most $d' = \frac{2Kd}{2+(K-1)d}$, and that this result is sharp. He conjectured (Question 4.4) that if the Hausdorff measure $\mathcal{H}^d (E)=0$, then $\mathcal{H}^{d'} (fE)=0$.

UT showed that Astala's conjecture is sharp in the class of all Hausdorff gauge functions. Lacey, Sawyer and UT jointly proved completely Astala's conjecture in all dimensions. The proof uses Astala's 1994 approach, geometric measure theory, and new weighted norm inequalities for Calder\'{o}n-Zygmund singular integral operators which cannot be deduced from the classical Muckenhoupt A_p theory.

These results are related to removability problems for various classes of quasiregular maps. I will mention sharp removability results for bounded K-quasiregular maps (i.e. the quasiconformal analogue of the classical Painlev'e problem) obtained jointly by Tolsa and UT.

I will further mention results related to another conjecture of Astala on Hausdorff dimension of quasicircles obtained jointly by Prause, Tolsa and UT. The talk will be self-contained.

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