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Erratum: W. S. Jassim, On the intersection of finitely generated subgroups of free groups, Rev. Mat. Univ. Compl. 9(1996), 67-84.

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Abstract

Attention is drawn to an error in W. S. Jassim's paper.

1 Introduction

Let F be a free group, and let H and K be finitely generated subgroups of F. Recently W. S. Jassim [2] claimed to obtain a certain upper bound for the rank of $H \cap K$ in terms of the ranks of H and K. (The author of this note has made similar erroneous claims, but all unpublished). Jassim's proof is not valid, and we will demonstrate this by giving a counterexample to his Lemma 2.4 in the next section.

In the period since Jassim's article appeared, a small amount of progress has been made in the problem of bounding the rank of $H \cap K$; see [3], [1], [4].

2 The lemma and the example

A crucial step in Jassim's argument is the following.

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Lemma 2.4 of [2]. *Let $\Gamma^*(H)$ be the core graph of finitely generated subgroup H of the free group F on generators a, b. If $\Gamma^*(H)$ has only two types of compatible branch points X_1 and X_2 then $x_1 = x_2$ where x_1 and x_2 are the number of compatible branch points of types X_1 and X_2 respectively and $\Gamma^*(H)$ has $2n = x_1 + x_2$ branch points.*

We wish to consider the example where the subgroup H is freely generated by $ba^{-1}b^{-1}$, $aba^{-1}b^{-1}a^{-1}$, $a^{-1}bab^{-1}aba^{-1}b^{-1}a^{-1}ba^{-1}b^{-1}a$. Here the core graph $\Gamma^*(H)$ is as depicted in Figure 1.

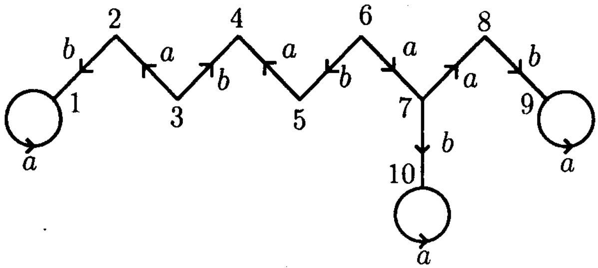

Figure 1. $\Gamma^*(H)$

We have an immersion, that is, a locally injective graph morphism, from this graph to the graph with one vertex and two edges labelled a and b. We take two copies of this immersion and form the pullback $\Gamma^*(H)\tilde{\times}\Gamma^*(H)$. For example, one of the components of this pullback is depicted in Figure 2.

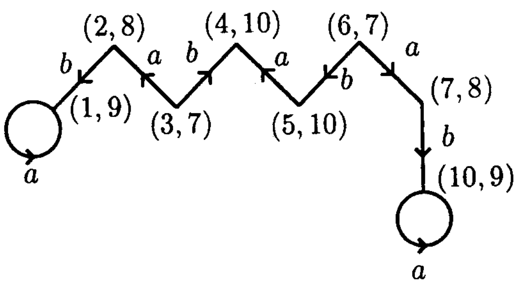

Figure 2. One component of $\Gamma^*(H)\tilde{\times}\Gamma^*(H)$

The definition of *compatible* given in [2] is open to some interpretation, but for the case $H = K$ it seems reasonable to understand that two branch points i and r of $\Gamma^*(H)$ are compatible of the same type if and only if (i, r) is a branch point in the core of the pullback $\Gamma^*(H)\tilde{\times}\Gamma^*(H)$. Since (1,9) and (10,9) are branch points in the core of $\Gamma^*(H)\tilde{\times}\Gamma^*(H)$ in our example, we see that the branch points 1, 9 and 10 are all compatible of the same type, which we call X_1. Since the branch point 7 has an edge labelled b leading into it, this branch point is clearly of a different type, which we call X_2. Thus $x_1 = 3$ and $x_2 = 1$, and these are not equal. Hence [2, Lemma 2.4] is not valid.

References

[1] Warren Dicks, *Equivalence of the strengthened Hanna Neumann conjecture and the amalgamated graph conjecture*, Invent. Math. **117** (1994), 373-389.

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[3] G. Tardos, *On the intersection of subgroups of a free group*, Invent. Math. **108** (1992), 29-36.

[4] G. Tardos, *Toward the Hanna Neumann conjecture using Dicks' method*, Invent. Math. **123** (1996), 95-104.

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