Erratum to

“Fundamental Groups of Some Special Quadric Arrangements”

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Meirav AMRAM and Mina TEICHER

Einstein Mathematics Institute
the Hebrew University
Jerusalem — Israel
ameirav@math.huji.ac.il

Department of Mathematics
Bar-Ilan University
Ramat-Gan — Israel
teicher@macs.biu.ac.il

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ABSTRACT

This erratum relates to our work “Fundamental groups of some special quadric arrangements”. The original Theorems 2.2, 2.5, 2.8 and Propositions 2.3(ii)(iii), 2.6(ii)(iii), 2.9(ii)(iii) have wrong results. They need to be rephrased. Corollaries 2.4 and 2.7 are incomplete, and they are extended. We add a new Corollary 2.10, which does not appear in the original paper. Proposition 3.1 has a wrong result and it is rephrased and reproved.

In Proposition 4.1 and its Corollary 4.2 a slight error has occurred: as the correct proofs in the paper show, the monodromy is a quadruple fulltwist.

Key words: Fundamental groups, complement of curve, quadric arrangement.

2000 Mathematics Subject Classification: 14H20, 14H30, 14Q05.

1. Wrong results

Theorems 2.2, 2.5, 2.8 and Propositions 2.3(ii)(iii), 2.6(ii)(iii), 2.9(ii)(iii) are wrong. We prove here new results 2.2, 2.5, 2.8, explaining the wrong results in the original version. Since Proposition 2.3(ii)(iii) presents the specific cases $n = 2, 3$ for the general n in Theorem 2.2, it can be combined with the New Theorem 2.2. In the same manner, we combine also Proposition 2.6(ii)(iii) with the New Theorem 2.5, and Proposition 2.9(ii)(iii) with the New Theorem 2.8.

Figures 2, 3, 4 (pages 264-266 in the paper) of the arrangements from Theorems 2.2, 2.5, 2.8 are replaced by Figures 1, 6, 9 respectively.

Figure 1 replaces Figure 2 from page 264 in the paper.

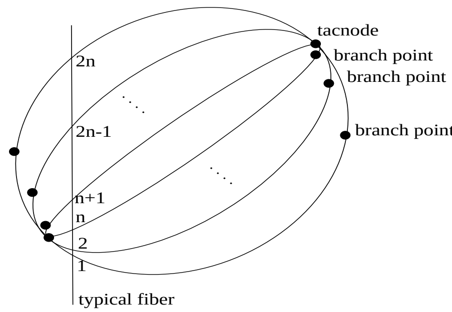


Figure 1 – The arrangement $\mathcal{A}_n$.

Theorem 2.2 and Proposition 2.3 (page 263-264 in the paper) are rephrased as one general statement as follows:

New Theorem 2.2. *The fundamental group $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_n)$ of $\mathcal{A}_n$ in $\mathbb{P}^2$ admits the presentation*

$$\pi_1(\mathbb{P}^2\backslash\mathcal{A}_n) \simeq \Big\langle a_1, a_2, \ldots, a_n \;\Big|\; (a_1 a_2 \cdots a_n)^2 = e \Big\rangle, \tag{1}$$

where $a_1, \ldots, a_n$ are meridians of $Q_1, \ldots, Q_n$, respectively.

Proof of New Theorem 2.2. In order to find the fundamental group $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_n)$, we need the algorithm of Moishezon-Teicher for global braid monodromy. Let us consider the following setting (Figure 2). S is an algebraic curve in $\mathbb{C}^2$, with $p = \deg(S)$. Let $\pi : \mathbb{C}^2 \to \mathbb{C}$ be a generic projection on the first coordinate. Define the fiber $K(x) = \{y \mid (x, y) \in S\}$ in S over a fixed point x, projected to the y-axis. Define $N = \{x \mid \#K(x) < p\}$ and $M' = \{s \in S \mid \pi_{|s}$ is not étale at $s\}$; note that $\pi(M') = N$. Let $\{A_j\}_{j=1}^q$ be the set of points of M' and $N = \{x_j\}_{j=1}^q$ their projection on the x-axis. Recall that π is generic, so we assume that $\#(\pi^{-1}(x) \cap M') = 1$ for every $x \in N$. Let E (resp. D) be a closed disk on the x-axis (resp. the y-axis), such that $M' \subset E \times D$ and $N \subset \mathrm{Int}(E)$. We choose $u \in \partial E$ a real point far enough from the set N, $x << u$ for every $x \in N$. Define $\mathbb{C}_u = \pi^{-1}(u)$ and number the points of $K = \mathbb{C}_u \cap S$ as $\{y_1, \ldots, y_p\}$.

We construct a g-base for the fundamental group $\pi_1(E - N, u)$. Take a set of paths $\{\gamma_j\}_{j=1}^q$ which connect u with the points $\{x_j\}_{j=1}^q$ of N. Now encircle each x_j with a small oriented counterclockwise circle c_j. Denote the path segment from u

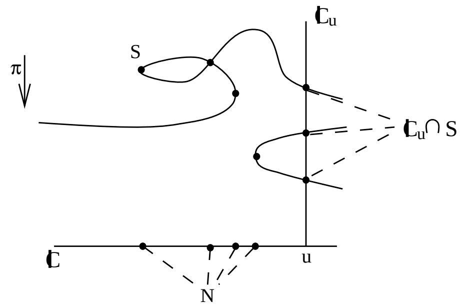


Figure 2 – General setting.

to the boundary of this circle as γ'_j. We define an element (a loop) in the g-base as $\delta_j = {\gamma'}_j c_j {\gamma'}_j^{-1}$. Let $B_p[D,K]$ be the braid group, and let $H_1,\ldots,H_{p-1}$ be its frame (for complete definitions, see [3, Section III.2]). The braid monodromy of S [1] is a map $\varphi : \pi_1(E-N,u) \to B_p[D,K]$ defined as follows: every loop in $E-N$ starting at u has liftings to a system of p paths in $(E-N)\times D$ starting at each point of $K = \{y_1,\ldots,y_p\}$. Projecting them to D we get p paths in D defining a motion $\{y_1(t),\ldots,y_p(t)\}$ (for $0 \le t \le 1$) of p points in D starting and ending at K. This motion defines a braid in $B_p[D,K]$. By the Artin Theorem [4], for $j = 1,\ldots,q$, there exists a halftwist $Z_j \in B_p[D,K]$ and $\epsilon_j \in \mathbb{Z}$, such that $\varphi(\delta_j) = Z_j^{\epsilon_j}$ ($\epsilon_j = 1, 2$ or 4 for an ordinary branch point, a node, or a tacnode respectively). We explain now how to get this Z_j.

Let A_j be a singular point in S and its projection by π to the x-axis is x_j. We choose a point x'_j next to x_j, such that $\pi^{-1}(x'_j)$ is a typical fiber, which intersects the two components which meet at A_j in two points, say a, b. We fix a skeleton $\xi_{x'_j}$ which connects a and b, and denote it as $< a, b >$ (in a case that more than two components meet at A_j, we consider a, b to be the two extreme components). The Lefschetz diffeomorphism Ψ (see [3]) allows us to get a resulting skeleton $(\xi_{x'_j})\Psi$ in the typical fiber $\mathbb{C}_u$. This one defines a motion of its two endpoints. This motion induces a halftwist $Z_j = \Delta < (\xi_{x'_j})\Psi >$. As above, $\varphi(\delta_j) = \Delta < (\xi_{x'_j})\Psi >^{\epsilon_j}$. The braid monodromy factorization associated to S is

$$\Delta_p^2 = \prod_{j=1}^{q} \varphi(\delta_j).$$

Before proving the theorem, we have to study the local monodromy around a common tacnode (of n quadrics) and the relation which is derived from this monodromy. We do it in the following propositions:

(i) The following proposition replaces Proposition 3.1 (page 267 in the paper). Proposition 3.1 is wrong and therefore it is rephrased and proved as follows.

New Proposition 3.1. *Let O be a singular point defined locally by $(y-x^2)(y-2x^2)\cdots(y-nx^2)$. Then the local monodromy around O is a double fulltwist on the disk (Figure 3 replaces Figure 5 from the paper, page 267).*

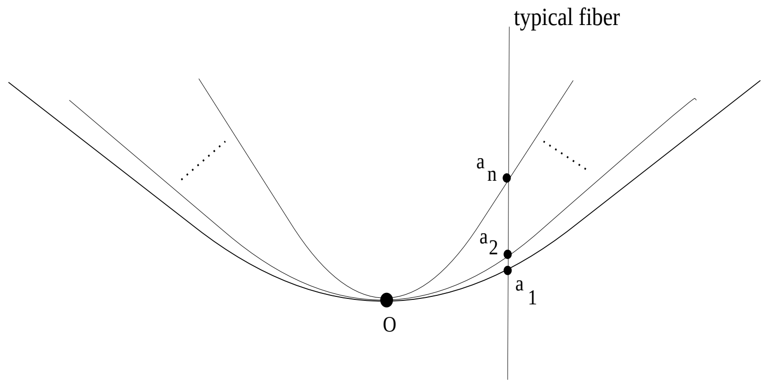


Figure 3 – The point O.

Proof. Take a loop $x = e^{2\pi i t}$ in $y = 0$, starting (and ending) at some base point and encircling the point O, $0 \leq t \leq 1$. Let $a_1, a_2, \ldots, a_n$ be the points of the curve in a typical fiber next to the fiber F_O.

When t is running from 0 to 1/2, the point a_1 is rotating around the other points in a fulltwist $e^{2\pi i}$, the point a_2 is rotating along a closed curve which bounds a disk, containing the trajectory of the point a_1, and so on. When t is running from 1/2 to 1, we have the same motion. This gives us the needed double fulltwist. ∎

(ii) The following proposition replaces the beginning of the proof of Theorem 2.2 (page 267 in the paper).

Proposition. *If a point O is one among two tacnodes between n quadrics (see Figure 1), then we have the relation*

$$(a_n \cdots a_2 a_1)^2 = (a_1 a_n \cdots a_2)^2 = \ldots = (a_{n-1} a_{n-2} \cdots a_1 a_n)^2,$$

and if O is a unique tacnode between n quadrics (see Figures 6 and 9), then we have the relation

$$(a_n \cdots a_2 a_1)^4 = (a_1 a_n \cdots a_2)^4 = \ldots = (a_{n-1} a_{n-2} \cdots a_1 a_n)^4.$$

The first part of the proposition is proved in the paper (pages 267-268). The second part is concluded easily from the first part and also from New Proposition 4.1 (see Section 2).

We are interested in the group $\pi_1(\mathbb{P}^2 \backslash \mathcal{A}_n)$. We start with $\mathcal{A}_1$ (a smooth quadric), and it is easy to see that the group $\pi_1(\mathbb{P}^2 \backslash \mathcal{A}_1)$ is $\mathbb{Z}_2$ (which is abelian).

The cases $n = 2, 3$ have wrong results in the paper (see Proposition 2.3(ii)(iii) in the paper). We prove new results here and conclude the general case $\pi_1(\mathbb{P}^2 \backslash \mathcal{A}_n)$. Let us consider the arrangement $\mathcal{A}_2$, see Figure 4 (replaces Figure 7 from the paper, page 269). We want to compute the braid monodromy factorization of $\mathcal{A}_2$. Consider

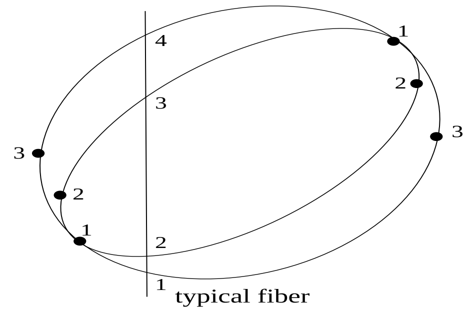


Figure 4 – The arrangement $\mathcal{A}_2$.

the above general setting (see Figure 2). By abuse of notation we denote the points y_1, y_2, y_3, y_4 of K as $1, 2, 3, 4$ respectively (see the typical fiber in Figure 4). For simplicity, we denote also the singular points as j instead of A_j (see the dotted points in Figure 4). Let $\{j\}_{j=1}^3$ be singular points of π_1 on the left side of a chosen typical fiber as follows: 1 is a tacnode (among two tacnodes in the arrangement), 2 and 3 are branch points of the quadrics.

We are looking for $\varphi(\delta_j)$ for $j = 1, 2, 3$. So we choose a g-base $\{\delta_j\}_{j=1}^3$ of $\pi_1(E - N, u)$, such that each δ_j is constructed from a path γ_j below the real line and a counterclockwise small circle around the points in N.

The diffeomorphisms which are induced from passing through branch points (such as $\Delta^{\frac{1}{2}}_{I_2\mathbb{R}} < k >$ and $\Delta^{\frac{1}{2}}_{I_4 I_2} < k >$) were defined in [4]. We recall the definition from [4]. Consider two typical fibers on the left and right sides of a branch point (locally defined by $y^2 - x = 0$). The typical fiber on the left side intersects the quadric in two complex points. Now, if we pass through this point to the right typical fiber (see Lefschetz diffeomorphism [3]), the two complex points move to the k'th place and rotate in a counterclockwise 90^o twist into the real axis and are numbered as $k, k+1$ (in the right typical fiber).

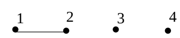
In order to construct the braid monodromy table, we find first the skeletons $\xi_{x'_j}$ related to the singular points (following Figure 4). Then we compute the local diffeomorphisms δ_j induced from the singular points. We fix them in the braid monodromy table below:

j	$\xi_{x'_j}$	ϵ_j	δ_j
1	$<1,2>$	4	$\Delta^2 <1,2>$
2	$<2,3>$	1	$\Delta^{\frac{1}{2}}_{I_2\mathbb{R}} <2>$
3	$<1,2>$	1	$\Delta^{\frac{1}{2}}_{I_4 I_2} <1>$

By Moishezon-Teicher algorithm [4], we compute the skeleton $(\xi_{x'_j})\Psi_{\gamma'_j}$ on the typical fiber for each j, by applying to the skeleton $\xi_{x'_j}$ the product $\prod\limits_{i=j-1}^{1} \delta_i$. The resulting braids are as follows.

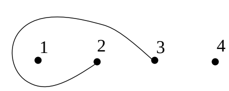
A braid related to the tacnode:

$$(\xi_{x'_1})\Psi_{\gamma'_1} = <1,2> = z_{1\;2},$$
$$\varphi(\delta_1) = Z^4_{1\;2},$$



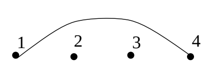
A braid related to the inner branch point:

$$(\xi_{x'_2})\Psi_{\gamma'_2} = <2,3> \Delta^2 <1,2> = z_{2\;3}^{Z^2_{1\;2}},$$
$$\varphi(\delta_2) = Z_{2\;3}^{Z^2_{1\;2}},$$



A braid related to the outer branch point:

$$(\xi_{x'_3})\Psi_{\gamma'_3} = <1,2> \Delta^{\frac{1}{2}}_{I_2\mathbb{R}} <2> \Delta^2 <1,2> = \bar{z}_{1\;4},$$
$$\varphi(\delta_3) = \bar{Z}_{1\;4},$$



One can conclude that when the singular points are on the left side of the typical fiber, we apply the diffeomorphisms from the table counterclockwise (exactly as done in [4] and as explained above). But if we have singular points on the right side of the typical fiber (see Figure 4), we apply the diffeomorphisms clockwise on the skeleton, namely $\prod\limits_{i=j-1}^{1} \delta_i^{-1}$. The resulting braids in this case are the complex conjugates of the ones which are derived as if we apply the diffeomorphisms counterclockwise (as we do for the left side), see works which use complex conjugates of braids, e.g. [2]. Since we are able to get the same relations for the fundamental group by simplifying the complex conjugates relations, we prefer the braids which are derived by applying counterclockwise diffeomorphisms. We remind that there are three singular points on the right side of the chosen typical fiber: 1 is a tacnode, 2 and 3 are branch points of the quadrics. The related monodromy table is:

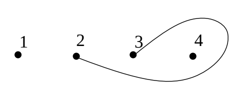
j	$\xi_{x'_j}$	ϵ_j	δ_j
1	$<3,4>$	4	$\Delta^2<3,4>$
2	$<2,3>$	1	$\Delta^{\frac{1}{2}}_{I_2\mathbb{R}}<2>$
3	$<1,2>$	1	$\Delta^{\frac{1}{2}}_{I_4 I_2}<1>$

A braid related to the tacnode:

$$(\xi_{x'_1})\Psi_{\gamma'_1} = <3,4> = z_{3\;4}$$
$$\varphi(\delta_1) = Z^4_{3\;4},$$

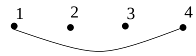
A braid related to the inner branch point:

$$(\xi_{x'_2})\Psi_{\gamma'_2} = <2,3> \Delta^2 <3,4> = z_{2\;3}^{Z^2_{3\;4}},$$
$$\varphi(\delta_2) = Z_{2\;3}{}^{Z^2_{3\;4}},$$



A braid related to the outer branch point:

$$(\xi_{x'_3})\Psi_{\gamma'_3} = <1,2> \Delta^{\frac{1}{2}}_{I_2\mathbb{R}} <2> \Delta^2 <3,4> = z_{1\;4},$$
$$\varphi(\delta_3) = Z_{1\;4},$$



According to the van Kampen Theorem [5], we get the following set of relations for $\pi_1(\mathbb{P}^2 \backslash \mathcal{A}_2)$:

$$\begin{aligned}
(a_3a_4)^2 &= (a_4a_3)^2, \\
a_2 &= a_4a_3a_4^{-1}, \\
a_1 &= a_4, \\
(a_1a_2)^2 &= (a_2a_1)^2, \\
a_3 &= a_2a_1a_2a_1^{-1}a_2^{-1}, \\
a_4 &= a_3a_2a_1a_2^{-1}a_3^{-1}, \\
a_4a_3a_2a_1 &= e \quad \text{(the projective relation)}.
\end{aligned}$$

By an easy simplification, we get

$$\pi_1(\mathbb{P}^2 \backslash \mathcal{A}_2) \simeq \left\langle a_1, a_2 \,\middle|\, (a_1a_2)^2 = e \right\rangle, \tag{2}$$

which is a big group (replaces Proposition 2.3(ii), page 264 in the paper).

Now, considering the arrangement $\mathcal{A}_3$ (see Figure 5), we can construct the tables of the global braid monodromy. We start with the table related to the singular points

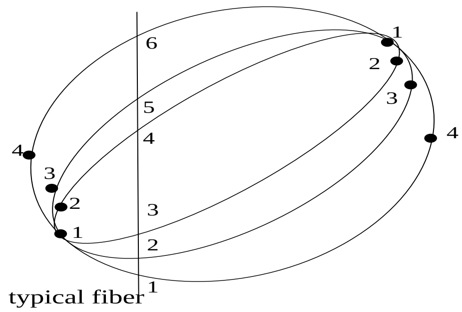


Figure 5 – The arrangement $\mathcal{A}_3$.

on the left side of the typical fiber:

j	$\xi_{x'_j}$	ϵ_j	δ_j
1	$< 1, 2, 3 >$	4	$\Delta^2 < 1, 2, 3 >$
2	$< 3, 4 >$	1	$\Delta^{\frac{1}{2}}_{I_2\mathbb{R}} < 3 >$
3	$< 2, 3 >$	1	$\Delta^{\frac{1}{2}}_{I_4I_2} < 2 >$
4	$< 1, 2 >$	1	$\Delta^{\frac{1}{2}}_{I_6I_4} < 1 >$

And we finish with the table related to the singular points on right side of the typical fiber:

j	$\xi_{x'_j}$	ϵ_j	δ_j
1	$< 4,5,6 >$	4	$\Delta^2 < 4,5,6 >$
2	$< 3,4 >$	1	$\Delta^{\frac{1}{2}}_{I_2\mathbb{R}} < 3 >$
3	$< 2,3 >$	1	$\Delta^{\frac{1}{2}}_{I_4 I_2} < 2 >$
4	$< 1,2 >$	1	$\Delta^{\frac{1}{2}}_{I_6 I_4} < 1 >$

Since we know already how to apply the algorithm of Moishezon-Teicher (using the tables) and how to apply the van Kampen Theorem (see [4] for the algorithm and [5] for the van Kampen Theorem), we can state the resulting presentation of $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_3)$ (after some simple simplifications):

$$\pi_1(\mathbb{P}^2\backslash\mathcal{A}_3) \simeq \left\langle a_1, a_2, a_3 \,\middle|\, (a_1a_2a_3)^2 = e \right\rangle . \tag{3}$$

It is easy to see that $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_3)$ is big (replaces Proposition 2.3(iii), page 264 in the paper).

In order to compute the group $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_n)$, we fix first a set of generators $a_1, \dots, a_{2n}$. Then we apply again the Moishezon-Teicher algorithm for the global monodromy. On each one of the sides of the typical fiber in Figure 1 we have one tacnode and n branch points. Since we have two tacnodes in the arrangement (see propositions above), we have relations (4) and (9). The relations concerning the right branch points are (5)–(8), and the relations concerning the left branch points are (10)–(13):

$$(a_{n+1}a_{n+2}\cdots a_{2n})^2 = (a_{2n}a_{n+1}\cdots a_{2n-1})^2 = \ldots = (a_{n+2}a_{n+3}\cdots a_{2n}a_{n+1})^2, \tag{4}$$

$$a_n = a_{2n}a_{2n-1}\cdots a_{n+2}a_{n+1}a_{n+2}^{-1}\cdots a_{2n-1}^{-1}a_{2n}^{-1}, \tag{5}$$

$$a_{n-1} = a_{2n}a_{2n-1}\cdots a_{n+3}a_{n+2}a_{n+3}^{-1}\cdots a_{2n-1}^{-1}a_{2n}^{-1}, \tag{6}$$

$$a_{n-2} = a_{2n}a_{2n-1}\cdots a_{n+4}a_{n+3}a_{n+4}^{-1}\cdots a_{2n-1}^{-1}a_{2n}^{-1}, \tag{7}$$

$\cdots\cdots$

$$a_1 = a_{2n}, \tag{8}$$

$$(a_1a_2\cdots a_n)^2 = (a_na_1\cdots a_{n-1})^2 = \ldots = (a_2a_3\cdots a_na_1)^2, \tag{9}$$

$$a_{n+1} = a_na_{n-1}\cdots a_1a_na_1^{-1}\cdots a_{n-1}^{-1}a_n^{-1}, \tag{10}$$

$$a_{n+2} = a_{n+1}a_n\cdots a_1a_{n-1}a_1^{-1}\cdots a_n^{-1}a_{n+1}^{-1}, \tag{11}$$

$$a_{n+3} = a_{n+2}a_{n+1}\cdots a_1a_{n-2}a_1^{-1}\cdots a_{n+1}^{-1}a_{n+2}^{-1}, \tag{12}$$

$\cdots\cdots$

$$a_{2n} = a_{2n-1}a_{2n-2}\cdots a_2a_1a_2^{-1}\cdots a_{2n-2}^{-1}a_{2n-1}^{-1}, \tag{13}$$

$$a_{2n}a_{2n-1}a_{2n-2}\cdots a_2a_1 = e, \quad \text{(the projective relation)}. \tag{14}$$

We substitute relations (5)–(8) in relations (10)–(13) and get for each one of them one of the following equations:

$$(a_{n+1}a_{n+2}\cdots a_{2n})^{-2} = (a_{2n}a_{n+1}\cdots a_{2n-1})^{-2} = \ldots = (a_{n+2}a_{n+3}\cdots a_{2n}a_{n+1})^{-2}.$$

These equations appear already in (4). Therefore relations (10)–(13) are redundant. Using relations (5)–(8), one can see that relations (4) and (9) are redundant.

By the same substitution, relation (14) is rewritten as $(a_{n+1}a_{n+2}\cdots a_{2n})^2 = e$. Hence, we get the presentation (1). □

Corollary 2.4 (page 264 in the paper) is incomplete and can be completed as follows:

New Corollary 2.4. *The group $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_n)$ is abelian for $n = 1$ and big for $n \geq 2$.*

Proof of New Corollary 2.4. We computed above the group $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_2)$. This group is isomorphic to $\mathbb{Z} * \mathbb{Z}_2$, which is known to be big. Since the group $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_2)$ is a quotient of $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_n)$ for $n \geq 2$, the groups $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_n)$ are big too. □

Remark 1.1. There is another way to prove that the groups $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_n)$ are big (for $n \geq 2$). These groups are isomorphic to $\mathbb{Z}^{n-1} * \mathbb{Z}_2$, which contain a non-abelian free subgroup.

Theorem 2.5 and Proposition 2.6 (page 264-265 in the paper) are rephrased as one general statement as follows:

New Theorem 2.5. *The fundamental group $\pi_1(\mathbb{P}^2\backslash\mathcal{B}_n)$ of $\mathcal{B}_n$ in $\mathbb{P}^2$ admits the presentation*

$$\pi_1(\mathbb{P}^2\backslash\mathcal{B}_n) \simeq \left\langle\ a_1, a_2, \ldots, a_n \ \middle|\ (a_1a_2\cdots a_n)^2 = e\ \right\rangle, \tag{15}$$

where $a_1, \ldots, a_n$ are meridians of $Q_1, \ldots, Q_n$, respectively. Figure 6 *replaces Figure 3 from page 265 in the paper.*

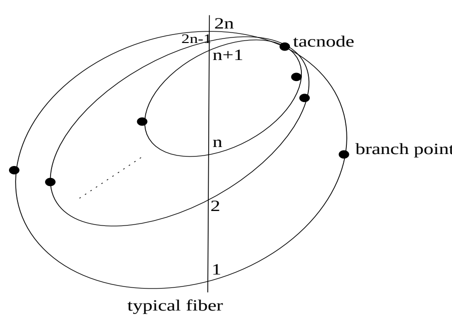


Figure 6 – The arrangement $\mathcal{B}_n$.

Proof of the New Theorem 2.5. We are interested in the group $\pi_1(\mathbb{P}^2 \backslash \mathcal{B}_n)$. We start with $\mathcal{B}_1$ (a smooth quadric), and it is easy to see that the group $\pi_1(\mathbb{P}^2 \backslash \mathcal{B}_1)$ is $\mathbb{Z}_2$ (which is abelian).

The cases $n = 2, 3$ have wrong results in the paper (see Proposition 2.6(ii)(iii) in the paper). We prove new results here and conclude the general case $\pi_1(\mathbb{P}^2 \backslash \mathcal{B}_n)$. Let us consider the arrangement $\mathcal{B}_2$, see Figure 7 (replaces Figure 8 from the paper, page 271). Let $\pi_1 : E \times D \to E$ be the projection to E. Take $u \in \partial E$, such that $\mathbb{C}_u$ is a

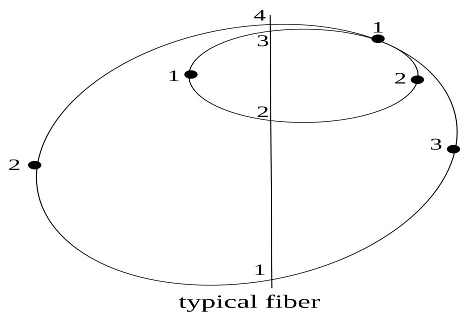


Figure 7 – The arrangement B_2.

typical fiber and let $K = \{1, 2, 3, 4\}$. For simplicity we denote also the singular points as j instead of A_j. Let $\{j\}_{j=1}^{2}$ be singular points of π_1 on the left side of the chosen typical fiber as follows: 1 is a branch point of the inner quadric, 2 is a branch point of the outer quadric. Let $N = \{x(j) = x_j \mid 1 \leq j \leq 2\}$, such that $N \subset E - \partial E, N \subset E$. We start with the monodromy table:

j	$\xi_{x'_j}$	ϵ_j	δ_j
1	$< 2, 3 >$	1	$\Delta^{\frac{1}{2}}_{I_2\mathbb{R}} < 2 >$
2	$< 1, 2 >$	1	$\Delta^{\frac{1}{2}}_{I_4 I_2} < 1 >$

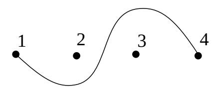
We have the following resulting braids:

A braid related to the inner branch point:

$$(\xi_{x'_1})\Psi_{\gamma'_1} =< 2, 3 >= z_{2\ 3},$$
$$\varphi(\delta_1) = Z_{2\ 3},$$

A braid related to the outer branch point:

$$(\xi_{x'_2})\Psi_{\gamma'_2} =< 1, 2 > \Delta^{\frac{1}{2}}_{I_2\mathbb{R}} < 2 >= z_{1\ 4}^{Z^2_{1\ 3}},$$
$$\varphi(\delta_2) = Z_{1\ 4}^{Z^2_{1\ 3}},$$



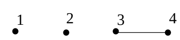
Now, in the same manner as explained before, we present the table which relates to the three singular points on the right side of the chosen typical fiber: 1 is a unique tacnode, 2 and 3 are branch points of the quadrics. The related monodromy table is:

j	$\xi_{x'_j}$	ϵ_j	δ_j
1	$< 3,4 >$	8	$\Delta^4 < 3,4 >$
2	$< 2,3 >$	1	$\Delta^{\frac{1}{2}}_{I_2\mathbb{R}} < 2 >$
3	$< 1,2 >$	1	$\Delta^{\frac{1}{2}}_{I_4 I_2} < 1 >$

By the algorithm from [4], we compute the braids:

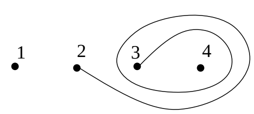
A braid related to the tacnode:

$$(\xi_{x'_1})\Psi_{\gamma'_1} = < 3,4 > = z_{3\;4},$$
$$\varphi(\delta_1) = Z^8_{3\;4},$$



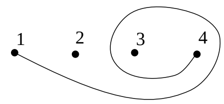
A braid related to the inner branch point:

$$(\xi_{x'_2})\Psi_{\gamma'_2} = < 2,3 > \Delta^4 < 3,4 > = z_{2\;3}^{Z^4_{3\;4}},$$
$$\varphi(\delta_2) = Z_{2\;3}{}^{Z^4_{3\;4}},$$



A braid related to the outer branch point:

$$(\xi_{x'_3})\Psi_{\gamma'_3} = < 1,2 > \Delta^{\frac{1}{2}}_{I_2\mathbb{R}} < 2 > \Delta^4 < 3,4 > = z_{1\;4}^{Z^2_{3\;4}},$$
$$\varphi(\delta_3) = Z_{1\;4}{}^{Z^2_{3\;4}},$$



The relations derived from these braids are as follows:

$$\begin{aligned}(a_3a_4)^4 &= (a_4a_3)^4,\\ a_2 &= a_4a_3a_4a_3a_4^{-1}a_3^{-1}a_4^{-1},\\ a_1 &= a_4a_3a_4a_3^{-1}a_4^{-1},\\ a_2 &= a_3,\\ a_1 &= a_3^{-1}a_4a_3,\\ a_4a_3a_2a_1 &= e, \quad \text{(the projective relation).}\end{aligned}$$

By an easy simplification, we get that $\pi_1(\mathbb{P}^2\backslash\mathcal{B}_2)$ is big (replaces Proposition 2.6(ii), page 265 in the paper): $\pi_1(\mathbb{P}^2\backslash\mathcal{B}_2) \simeq \langle a_1, a_2 \,\big|\, (a_1a_2)^2 = e\rangle$ (as in (2)).

Now, considering the arrangement $\mathcal{B}_3$ (see Figure 8), we can construct the tables of the global braid monodromy. We start with the table related to the singular points

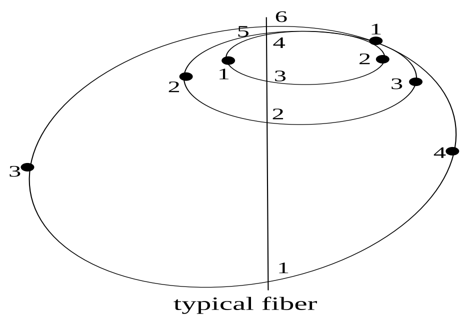


Figure 8 – The arrangement $\mathcal{B}_3$.

on the left side of the typical fiber:

j	$\xi_{x'_j}$	ϵ_j	δ_j
1	$<3,4>$	1	$\Delta^{\frac{1}{2}}_{I_2\mathbb{R}}<3>$
2	$<2,3>$	1	$\Delta^{\frac{1}{2}}_{I_4I_2}<2>$
3	$<1,2>$	1	$\Delta^{\frac{1}{2}}_{I_6I_4}<1>$

And we finish with the table related to the singular points on the right side of the typical fiber:

j	$\xi_{x'_j}$	ϵ_j	δ_j
1	$<4,5,6>$	8	$\Delta^4 <4,5,6>$
2	$<3,4>$	1	$\Delta^{\frac{1}{2}}_{I_2\mathbb{R}} <3>$
3	$<2,3>$	1	$\Delta^{\frac{1}{2}}_{I_4 I_2} <2>$
4	$<1,2>$	1	$\Delta^{\frac{1}{2}}_{I_6 I_4} <1>$

Again by the algorithm of Moishezon-Teicher [4] we compute the braids and by the van Kampen Theorem [5] and easy simplifications we get that $\pi_1(\mathbb{P}^2 \setminus \mathcal{B}_3)$ is big (replaces Proposition 2.6(iii), page 265 in the paper):

$$\pi_1(\mathbb{P}^2 \setminus \mathcal{B}_3) \simeq \left\langle a_1, a_2, a_3 \,\middle|\, (a_1 a_2 a_3)^2 = e \right\rangle, \quad \text{as in (3).}$$

In order to compute the group $\pi_1(\mathbb{P}^2 \setminus \mathcal{B}_n)$, we fix a set of generators $a_1, \dots, a_{2n}$. Then we apply again the Moishezon-Teicher algorithm for the global monodromy. On the left side of the typical fiber (see Figure 6) we have n branch points, and on the right side of it we have one tacnode and n branch points. The relations concerning the left points are:

$$a_n = a_{n+1}, \tag{16}$$
$$a_{n-1} = a_{n+1}^{-1} a_{n+2} a_{n+1}, \tag{17}$$
$$\cdots\cdots$$
$$a_1 = a_{n+1}^{-1} a_{n+2}^{-1} \cdots a_{2n-1}^{-1} a_{2n} a_{2n-1} \cdots a_{n+2} a_{n+1}, \tag{18}$$

and the relations concerning the right points are:

$$(a_{n+1} a_{n+2} \cdots a_{2n})^4 = (a_{2n} a_{n+1} \cdots a_{2n-1})^4 = \ldots = (a_{n+2} a_{n+3} \cdots a_{2n} a_{n+1})^4, \tag{19}$$
$$\begin{aligned} a_n = {} & a_{2n} a_{2n-1} \cdots a_{n+2} a_{n+1} a_{2n} a_{2n-1} \cdots a_{n+2} a_{n+1} a_{n+2}^{-1} \cdots a_{2n-1}^{-1} a_{2n}^{-1} \cdot \\ & \cdot a_{n+1}^{-1} a_{n+2}^{-1} \cdots a_{2n-1}^{-1} a_{2n}^{-1}, \end{aligned} \tag{20}$$
$$\begin{aligned} a_{n-1} = {} & a_{2n} a_{2n-1} \cdots a_{n+2} a_{n+1} a_{2n} a_{2n-1} \cdots a_{n+3} a_{n+2} a_{n+3}^{-1} \cdots a_{2n-1}^{-1} a_{2n}^{-1} \cdot \\ & \cdot a_{n+1}^{-1} a_{n+2}^{-1} \cdots a_{2n-1}^{-1} a_{2n}^{-1}, \end{aligned} \tag{21}$$
$$\cdots\cdots$$
$$a_1 = a_{2n} a_{2n-1} \cdots a_{n+2} a_{n+1} a_{2n} a_{n+1}^{-1} a_{n+2}^{-1} \cdots a_{2n-1}^{-1} a_{2n}^{-1}. \tag{22}$$

The projective relation is:

$$a_{2n} a_{2n-1} a_{2n-2} \cdots a_2 a_1 = e.$$

We substitute relation (16) in relation (20) and get

$$(a_{n+1} a_{2n} a_{2n-1} \cdots a_{n+2})^2 = (a_{2n} a_{2n-1} \cdots a_{n+1})^2.$$

Substituting relation (17) in relation (21), we get

$$(a_{n+1}a_{2n}a_{2n-1}\cdots a_{n+2})^2 = (a_{n+2}a_{n+1}a_{2n}\cdots a_{n+3})^2.$$

Repeating this procedure, we get for each one of the substitutions (the last step is the substitution of (18) in (22)) one of the following equations:

$$(a_{2n}a_{2n-1}\cdots a_{n+1})^2 = (a_{n+1}a_{2n}\cdots a_{n+2})^2 = \ldots = (a_{2n-1}\cdots a_{n+1}a_{2n})^2.$$

Therefore relation (19) is redundant.

The projective relation is rewritten as $(a_{n+1}a_{n+2}\cdots a_{2n})^2 = e$ (by the same substitutions). Hence we get the presentation (15). $\square$

Corollary 2.7 (page 265 in the paper) is incomplete and can be completed as follows:

New Corollary 2.7. *The group $\pi_1(\mathbb{P}^2\backslash\mathcal{B}_n)$ is abelian for $n = 1$ and big for $n \geq 2$.*

Proof of New Corollary 2.7. As in New Corollary 2.4. $\square$

Theorem 2.8 and Proposition 2.9 (page 265-266 in the paper) are rephrased as one general statement as follows:

New Theorem 2.8. *The fundamental group $\pi_1(\mathbb{P}^2\backslash\mathcal{C}_n)$ of $\mathcal{C}_n$ in $\mathbb{P}^2$ admits the presentation*

$$\pi_1(\mathbb{P}^2\backslash\mathcal{C}_n) \simeq \left\langle a_1,\ldots,a_n \;\middle|\; \begin{array}{ll} [a_i,a_j] = e & 2 \leq i,j \leq n, \quad i \neq j \\ (a_1a_k)^2 = (a_ka_1)^2 & 2 \leq k \leq n \\ (a_1a_2\cdots a_n)^2 = e & \end{array} \right\rangle, \tag{23}$$

where $a_1,\ldots,a_n$ are meridians of $Q_1,\ldots,Q_n$, respectively. Figure 9 replaces Figure 4 from page 266 in the paper.

Proof of the New Theorem 2.8. We are interested in the group $\pi_1(\mathbb{P}^2\backslash\mathcal{C}_n)$. We start with $\mathcal{C}_1$ (a smooth quadric), and it is easy to see that the group $\pi_1(\mathbb{P}^2\backslash\mathcal{C}_1)$ is $\mathbb{Z}_2$ (which is abelian).

The cases $n = 2,3$ have wrong results in the paper (see Proposition 2.9(ii)(iii) in the paper). We prove new results here and conclude the general case $\pi_1(\mathbb{P}^2\backslash\mathcal{C}_n)$, which is stated in the theorem. Let us consider the arrangement $\mathcal{C}_2$. The arrangements $\mathcal{B}_2$ and $\mathcal{C}_2$ look the same, see Figure 7 (replaces Figure 8 from the paper, page 271). Therefore, $\pi_1(\mathbb{P}^2\backslash\mathcal{C}_2) \simeq \langle a_1,a_2 \,|\, (a_1a_2)^2 = e\rangle$ (as in (2)), and it is big (replaces Proposition 2.9(ii), page 266 in the paper).

The arrangement $\mathcal{C}_3$ is depicted in Figure 10. For the arrangement $\mathcal{C}_3$ we first fix local typical fibers. Let $\pi_1 : E \times D \to E$ be the projection to E. Let $\{A_j\}_{j=1}^{12}$ be singular points of π_1 as follows (the computations for $\mathcal{C}_3$ involve some sophisticated description of the numeration in the fiber, so in order to prevent confusion , we denote

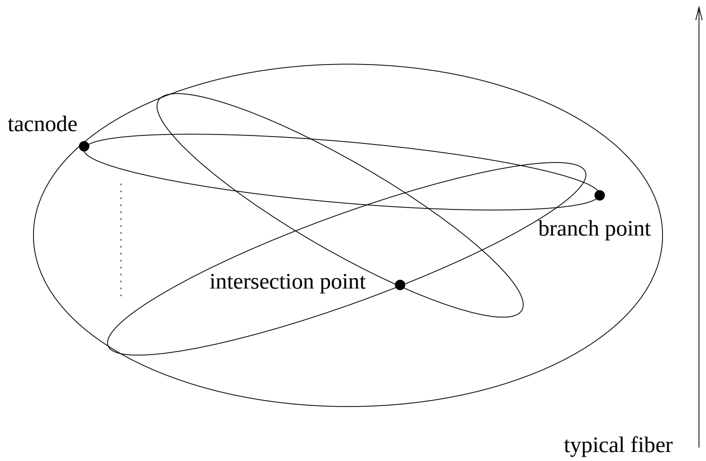


Figure 9 – The arrangement $\mathcal{C}_n$.

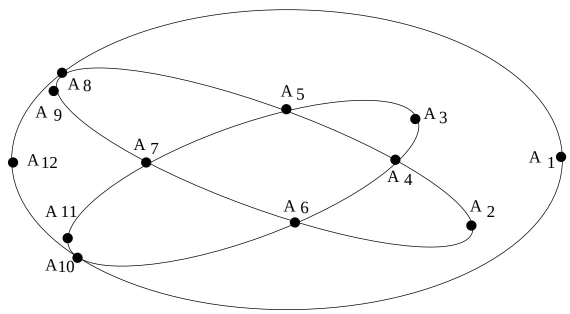


Figure 10 – The arrangement $\mathcal{C}_3$.

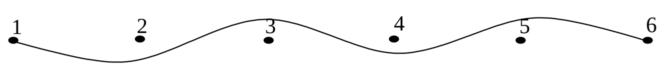
the singular points as originally stated, i.e. as A_j): $A_1, A_2, A_3, A_9, A_{11}, A_{12}$ are branch points, A_4, A_5, A_6, A_7 are intersections of the conics, and A_8, A_{10} are tacnodes. Let $K = \{1, 2, 3, 4, 5, 6\}$, such that all of them are real. In order to compute the braid monodromy, we have to move from one typical fiber to another (starting from the very left one, on the left side of A_{12}) till we arrive to the typical fiber which is positioned on the right side of the configuration (i.e. on the right side of A_1). The points $A_4, A_5, \ldots, A_{12}$ induce already known diffeomorphisms (see [4] and the tables above). Now, when we pass through the point A_3 to the next typical fiber on its right side, the points 4 and 5 in K turn to be complex (rotating counterclockwise in 90^o out of the axis), say $5+i, 5-i$ (while letting the point 6 move and replace the previous real position of 4 on the axis). So we have now a local typical fiber on the right side of A_3 with $K' = \{1, 2, 3, 4, 5+i, 5-i\}$. Passing through the point A_2

to the next local typical fiber on its right side, the points 2 and 3 in K' turn to be complex, say $4+i/2, 4-i/2$ (while letting the point 4 move and replace the previous real position of 2 on the axis). So we have now a local typical fiber on the right side of A_2 with $K'' = \{1, 2, 4+i/2, 4-i/2, 5+i, 5-i\}$. The last step is to pass through A_1 to the final typical fiber. This causes also turning 1 and 2 to complex points, say $3+i/4, 3-i/4$. So $K''' = \{3+i/4, 3-i/4, 4+i/2, 4-i/2, 5+i, 5-i\}$. Now, since we are interested in the fundamental group $\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_3)$, we let the complex points rotating in a counterclockwise manner back into their right positions in the axis. In this way it is much convenient to deduce through the van Kampen Theorem [5] the relations which relate to $\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_3)$ (the resulting braids appear below, involving an axis with six real points). The table of the monodromy is the following:

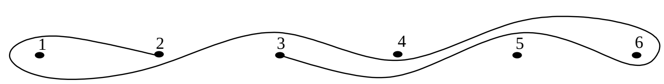
j	$\xi_{x'_j}$	ϵ_j	δ_j
1	P_1	1	$\Delta^{\frac{1}{2}}_{I_4 I_6} < 1 >$
2	P_2	1	$\Delta^{\frac{1}{2}}_{I_2 I_4} < 2 >$
3	P_4	1	$\Delta^{\frac{1}{2}}_{\mathbb{R} I_2} < 4 >$
4	$< 3, 4 >$	2	$\Delta < 3, 4 >$
5	$< 4, 5 >$	2	$\Delta < 4, 5 >$
6	$< 2, 3 >$	2	$\Delta < 2, 3 >$
7	$< 3, 4 >$	2	$\Delta < 3, 4 >$
8	$< 5, 6 >$	8	$\Delta^4 < 5, 6 >$
9	$< 4, 5 >$	1	$\Delta^{\frac{1}{2}}_{I_2 \mathbb{R}} < 4 >$
10	$< 1, 2 >$	8	$\Delta^4 < 1, 2 >$
11	$< 2, 3 >$	1	$\Delta^{\frac{1}{2}}_{I_4 I_2} < 2 >$
12	$< 1, 2 >$	1	$\Delta^{\frac{1}{2}}_{I_6 I_4} < 1 >$

We note that P_1 (resp. P_2, P_4) is the skeleton connecting the points 1 and 2 (resp. 2 and 3, 4 and 5) when they become complex, see notion and use in [4].

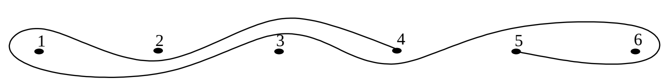
$\varphi(\delta_1) = Z_{1\,6}^{Z^2_{1\,5} Z^2_{1\,3}}$



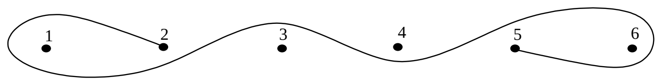
$\varphi(\delta_2) = Z_{2\,3}^{Z^{-2}_{1\,2} Z^{-2}_{3\,6} Z^{-2}_{5\,6}}$



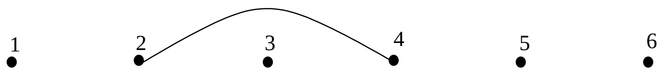
$\varphi(\delta_3) = Z_{4\,5}^{Z^{-2}_{5\,6} Z^{-2}_{1\,4} Z^{2}_{1\,3}}$



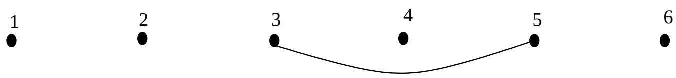
$\varphi(\delta_4) = (Z^2_{2\;5})^{Z^2_{2\;3}Z^{-2}_{5\;6}Z^{-2}_{1\;2}}$



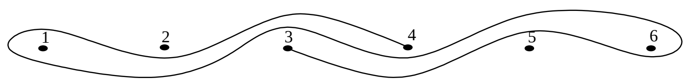
$\varphi(\delta_5) = \bar{Z}^2_{2\;4}$



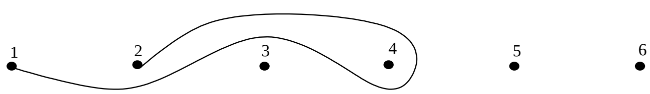
$\varphi(\delta_6) = Z^2_{3\;5}$



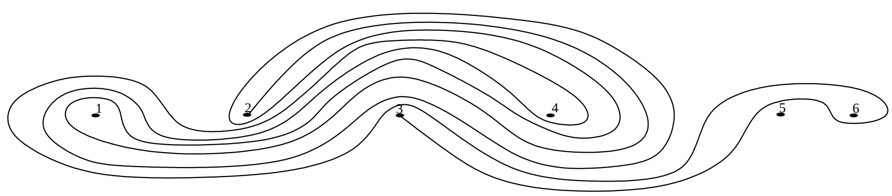
$\varphi(\delta_7) = (Z^2_{3\;4})^{Z^2_{1\;4}Z^{-1}_{3\;4}Z^{-2}_{3\;6}Z^{-2}_{5\;6}}$



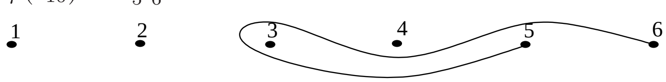
$\varphi(\delta_8) = (Z^8_{1\;2})^{\bar{Z}^2_{2\;4}}$



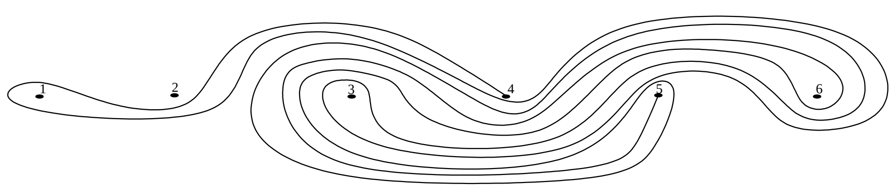
$\varphi(\delta_9) = \tilde{Z}_{2\;3}$



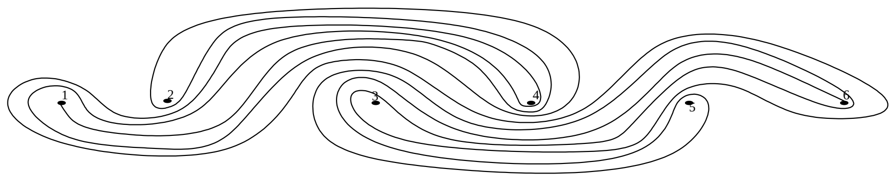
$\varphi(\delta_{10}) = (Z^8_{5\;6})^{Z^2_{3\;5}}$



$\varphi(\delta_{11}) = \tilde{Z}_{4\;5}$



$\varphi(\delta_{12}) = \tilde{Z}_{1\ 6}$



The group $\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_3)$ is then generated by $a_1, \ldots, a_6$ and admits the following presentation:

$$
\begin{aligned}
& a_3 a_1 a_3^{-1} = a_5^{-1} a_6 a_5, && (24)\\
& a_1^{-1} a_2 a_1 = a_3^{-1} a_5^{-1} a_6^{-1} a_5 a_3 a_5^{-1} a_6 a_5 a_3, &&\\
& a_3 a_1^{-1} a_3^{-1} a_4 a_3 a_1 a_3^{-1} = a_5^{-1} a_6^{-1} a_5 a_6 a_5, && (25)\\
& [a_3 a_1^{-1} a_2 a_1 a_3^{-1}, a_5^{-1} a_6^{-1} a_5 a_6 a_5] = e, &&\\
& [a_3^{-1} a_2 a_3, a_4] = e, && (26)\\
& [a_3, a_5] = e, && (27)\\
& [a_1^{-1} a_3^{-1} a_4 a_3 a_1, a_3^{-1} a_5^{-1} a_6^{-1} a_5 a_3 a_5^{-1} a_6 a_5 a_3] = e, &&\\
& (a_1 \cdot a_3^{-1} a_4 a_3 a_2 a_3^{-1} a_4^{-1} a_3)^4 = (a_3^{-1} a_4 a_3 a_2 a_3^{-1} a_4^{-1} a_3 \cdot a_1)^4, &&\\
& a_3 = a_5^{-1} a_6 a_5 a_3 a_1^{-1} a_3^{-1} a_4^{-1} a_3 a_1 a_3^{-1} a_4 a_3 a_2 a_3^{-1} a_4^{-1} a_3 a_1 a_3^{-1} a_4 a_3 a_2 \cdot &&\\
& \qquad \cdot a_3^{-1} a_4^{-1} a_3 a_1^{-1} a_3^{-1} a_4 a_3 a_2^{-1} a_3^{-1} a_4^{-1} a_3 a_1^{-1} a_3^{-1} a_4 a_3 a_1 a_3^{-1} a_5^{-1} a_6^{-1} a_5, &&\\
& (a_6 \cdot a_5 a_3 a_5 a_3^{-1} a_5^{-1})^4 = (a_5 a_3 a_5 a_3^{-1} a_5^{-1} \cdot a_6)^4, &&\\
& a_1^{-1} a_5 a_1 = a_3^{-1} a_5^{-1} a_6^{-1} a_3^{-1} a_6 a_5 a_6 a_5 a_6^{-1} a_5^{-1} a_6^{-1} a_3 a_6 a_5 a_3, &&\\
& a_1^{-1} a_3^{-1} a_4^{-1} a_3 a_1 a_3^{-1} a_4 a_3 a_2 a_3^{-1} a_4^{-1} a_3 a_1 a_3^{-1} a_4 a_3 a_2^{-1} a_3^{-1} a_4^{-1} a_3 a_1^{-1} a_3^{-1} a_4 a_3 a_1, &&\\
& = a_3^{-1} a_5^{-1} a_6^{-1} a_5 a_3^{-1} a_5^{-1} a_6 a_5 a_3 a_5 a_3^{-1} a_5^{-1} a_6 a_5 a_3 a_5^{-1} a_3^{-1} a_5^{-1} a_6^{-1} a_5 a_3 a_5^{-1} a_6 a_5 a_3, &&\\
& a_6 a_5 a_4 a_3 a_2 a_1 = e \ \ \text{(the projective relation)}. &&
\end{aligned}
$$

Substituting (24) in (25), we get $a_4 = a_5$. Relation (26) is rewritten as $[a_2, a_5] = e$ (see relation (27)). So with $a_4 = a_5$ and $[a_2, a_5] = [a_3, a_5] = e$, we can get a simplified presentation as follows:

$$
\begin{aligned}
&a_3a_1a_3^{-1} = a_5^{-1}a_6a_5, && (28)\\
&a_1^{-1}a_2a_1 = a_3^{-1}a_5^{-1}a_6^{-1}a_3a_6a_5a_3, && (29)\\
&[a_3a_1^{-1}a_2a_1a_3^{-1}, a_5^{-1}a_6^{-1}a_5a_6a_5] = e, &&\\
&[a_2, a_5] = e, &&\\
&[a_3, a_5] = e, &&\\
&[a_1^{-1}a_5a_1, a_3^{-1}a_5^{-1}a_6^{-1}a_3a_6a_5a_3] = e, && (30)\\
&(a_1a_2)^4 = (a_2a_1)^4, &&\\
&a_3 = a_6a_5a_3a_1^{-1}a_5^{-1}a_1a_2a_1a_2a_1^{-1}a_2^{-1}a_1^{-1}a_5a_1a_3^{-1}a_5^{-1}a_6^{-1}, && (31)\\
&(a_5a_6)^4 = (a_6a_5)^4, &&\\
&a_1^{-1}a_5^{-1}a_1a_2a_1a_2^{-1}a_1^{-1}a_5a_1 = a_3^{-1}a_5^{-1}a_6^{-1}a_3^{-1}a_6a_5a_6a_5^{-1}a_6^{-1}a_3a_6a_5a_3, && (32)\\
&a_1^{-1}a_5a_1 = a_3^{-1}a_5^{-1}a_6^{-1}a_3^{-1}a_6a_5a_6a_5a_6^{-1}a_5^{-1}a_6^{-1}a_3a_6a_5a_3, && (33)\\
&a_6a_5^2a_3a_2a_1 = e \quad \text{(the projective relation).} &&
\end{aligned}
$$

Substituting (29) in (31), we get $(a_1a_2)^2 = (a_2a_1)^2$. With (29) and then with (28), we can rewrite relation (32) as $(a_5a_6)^2 = (a_6a_5)^2$. Relation (33) is rewritten as $a_1^{-1}a_5a_1 = a_3^{-1}a_6a_5a_6^{-1}a_3$, using relation (30). So we have the following relations:

$$
\begin{aligned}
a_3a_1a_3^{-1} &= a_5^{-1}a_6a_5, && (34)\\
a_1^{-1}a_2a_1 &= a_3^{-1}a_5^{-1}a_6^{-1}a_3a_6a_5a_3, && (35)\\
[a_3a_1^{-1}a_2a_1a_3^{-1}, a_6a_5a_6^{-1}] &= e, && (36)\\
[a_2, a_5] &= e, &&\\
[a_3, a_5] &= e, &&\\
[a_1^{-1}a_5a_1, a_3^{-1}a_5^{-1}a_6^{-1}a_3a_6a_5a_3] &= e, && (37)\\
(a_1a_2)^2 &= (a_2a_1)^2, &&\\
(a_5a_6)^2 &= (a_6a_5)^2, &&\\
a_1^{-1}a_5a_1 &= a_3^{-1}a_6a_5a_6^{-1}a_3, && (38)\\
a_6a_5^2a_3a_2a_1 &= e. &&
\end{aligned}
$$

Now, substituting (35) in (36) and in (37), we get $[a_3, a_5] = e$ and $[a_2, a_5] = e$ respectively, so (36) and (37) are redundant. At this stage, we replace a_1 with $a_3^{-1}a_5^{-1}a_6a_5a_3$ (using (34)) in (38) to get $(a_5a_6)^2 = (a_6a_5)^2$, and therefore (38) is redundant. With the same substitution, from (35) we get $a_2 = a_3$. We replace a_2 with a_3. And these are the resulting relations:

$$\begin{aligned} a_1 &= a_3^{-1}a_5^{-1}a_6a_5a_3, \\ [a_3, a_5] &= e, \\ (a_1a_3)^2 &= (a_3a_1)^2, \\ (a_5a_6)^2 &= (a_6a_5)^2, \\ a_6a_5^2a_3^2a_1 &= e. \end{aligned}$$

Now we continue replacing a_1 with $a_3^{-1}a_5^{-1}a_6a_5a_3$ and we get the final presentation:

$$\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_3) \simeq \left\langle a_3, a_5, a_6 \;\middle|\; \begin{array}{l} [a_3, a_5] = e \\ (a_3a_6)^2 = (a_6a_3)^2 \\ (a_5a_6)^2 = (a_6a_5)^2 \\ (a_6a_5a_3)^2 = e \end{array} \right\rangle,$$

where a_3, a_5, a_6 are meridians of the three quadrics in Figure 10 (replaces Proposition 2.9(iii), page 266 in the paper).

Now let us consider the arrangement $\mathcal{C}_4$ (Figure 11). Let us fix a base point for

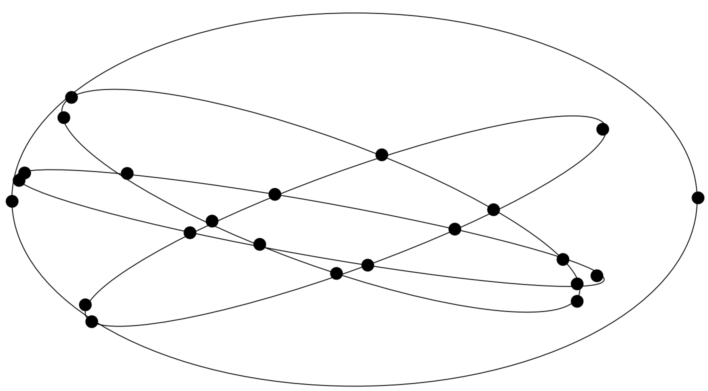


Figure 11 – The arrangement $\mathcal{C}_4$.

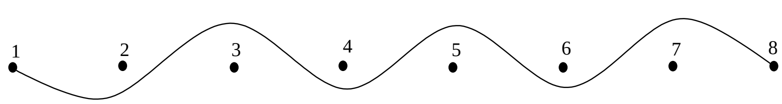
the fundamental group $\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_4)$. The setting in this case is similar to the one of $\mathcal{C}_3$, so we choose the typical fiber on the right side of the configuration, in which the points of the curve are complex. The base point is chosen to be on this fiber.

The braid monodromy table is:

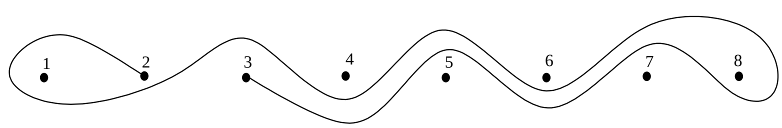
j	$\xi_{x'_j}$	ϵ_j	δ_j
1	P_1	1	$\Delta^{\frac{1}{2}}_{I_6 I_8} < 1 >$
2	P_2	1	$\Delta^{\frac{1}{2}}_{I_4 I_6} < 2 >$
3	P_4	1	$\Delta^{\frac{1}{2}}_{I_2 I_4} < 4 >$
4	P_6	1	$\Delta^{\frac{1}{2}}_{\mathbb{R} I_2} < 6 >$
5	$< 5, 6 >$	2	$\Delta < 5, 6 >$
6	$< 4, 5 >$	2	$\Delta < 4, 5 >$
7	$< 3, 4 >$	2	$\Delta < 3, 4 >$
8	$< 4, 5 >$	2	$\Delta < 4, 5 >$
9	$< 2, 3 >$	2	$\Delta < 2, 3 >$
10	$< 5, 6 >$	2	$\Delta < 5, 6 >$
11	$< 6, 7 >$	2	$\Delta < 6, 7 >$
12	$< 5, 6 >$	2	$\Delta < 5, 6 >$
13	$< 3, 4 >$	2	$\Delta < 3, 4 >$
14	$< 4, 5 >$	2	$\Delta < 4, 5 >$
15	$< 3, 4 >$	2	$\Delta < 3, 4 >$
16	$< 5, 6 >$	2	$\Delta < 5, 6 >$
17	$< 7, 8 >$	8	$\Delta^4 < 7, 8 >$
18	$< 6, 7 >$	1	$\Delta^{\frac{1}{2}}_{I_2 \mathbb{R}} < 6 >$
19	$< 1, 2 >$	8	$\Delta^4 < 1, 2 >$
20	$< 2, 3 >$	1	$\Delta^{\frac{1}{2}}_{I_4 I_2} < 2 >$
21	$< 3, 4 >$	8	$\Delta^4 < 3, 4 >$
22	$< 2, 3 >$	1	$\Delta^{\frac{1}{2}}_{I_6 I_4} < 2 >$
23	$< 1, 2 >$	1	$\Delta^{\frac{1}{2}}_{I_8 I_6} < 1 >$

Examples of braid monodromy were computed already above and also in the basic paper [4]. Therefore we skip the long computations in this part and present the resulting braids:

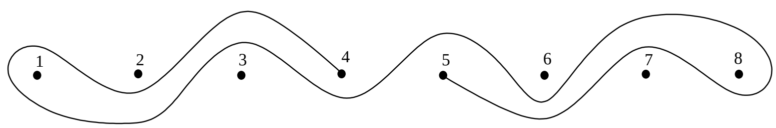
1. Branch point:



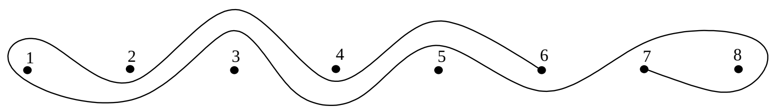
2. Branch point:



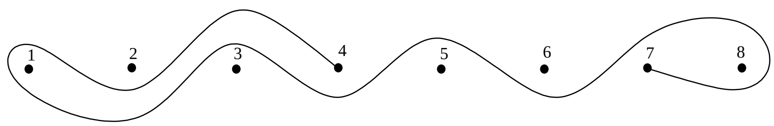
3. Branch point:



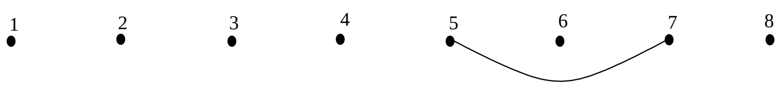
4. Branch point:



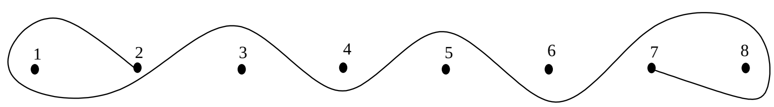
5. Intersection point:



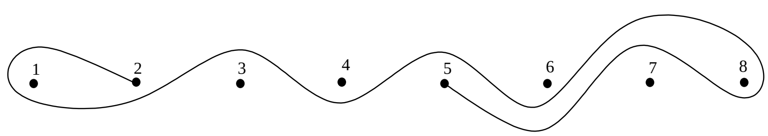
6. Intersection point:



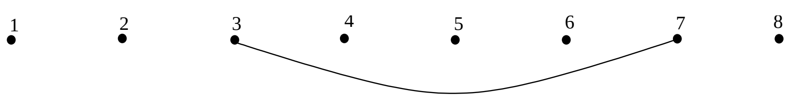
7. Intersection point:



8. Intersection point:

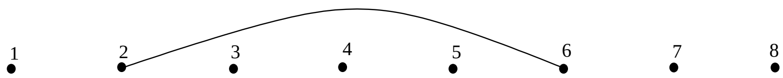


9. Intersection point:



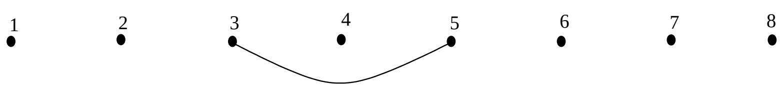
10. Intersection point:

11. Intersection point:

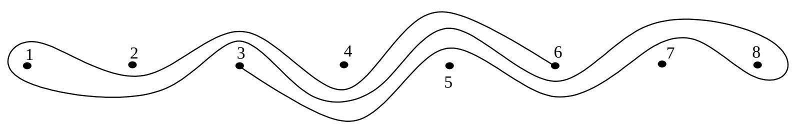


12. Intersection point:

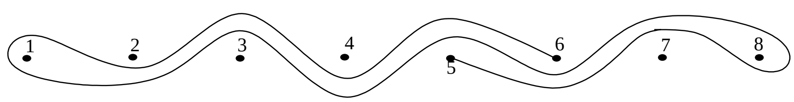
13. Intersection point:



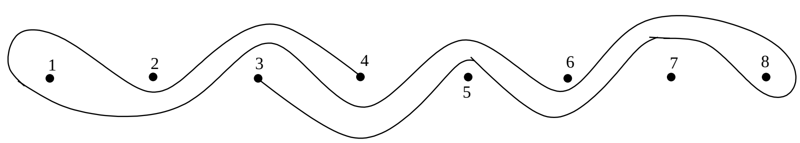
14. Intersection point:



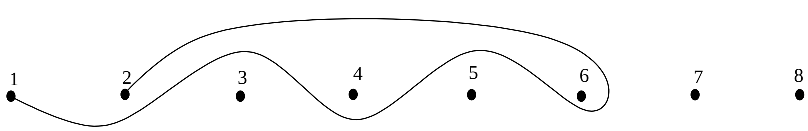
15. Intersection point:



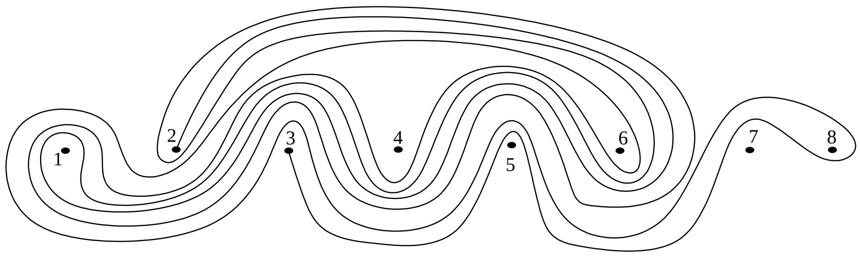
16. Intersection point:



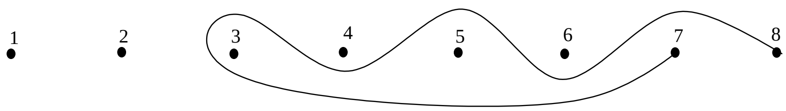
17. Tacnode:



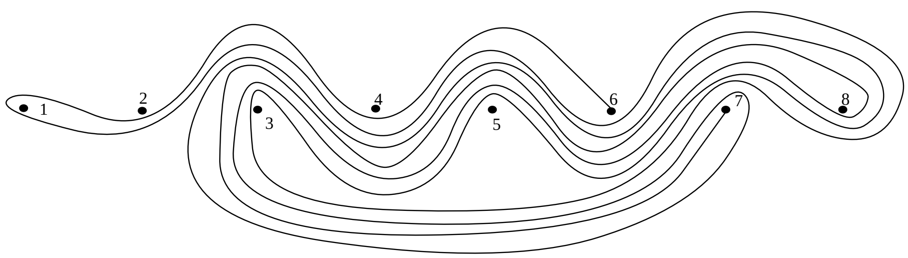
18. Branch point:



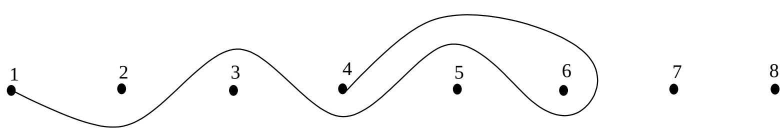
19. Tacnode:



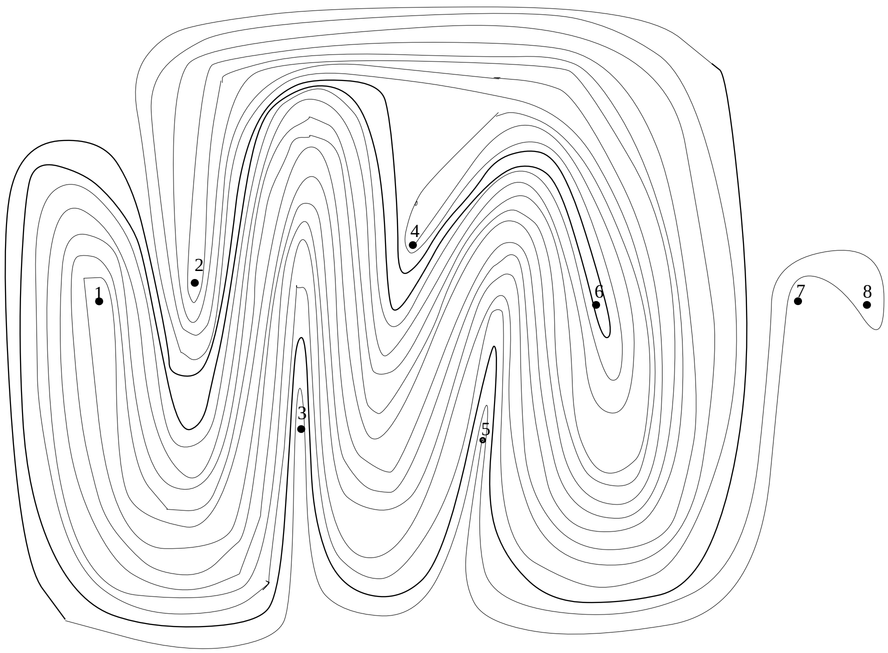
20. Branch point:



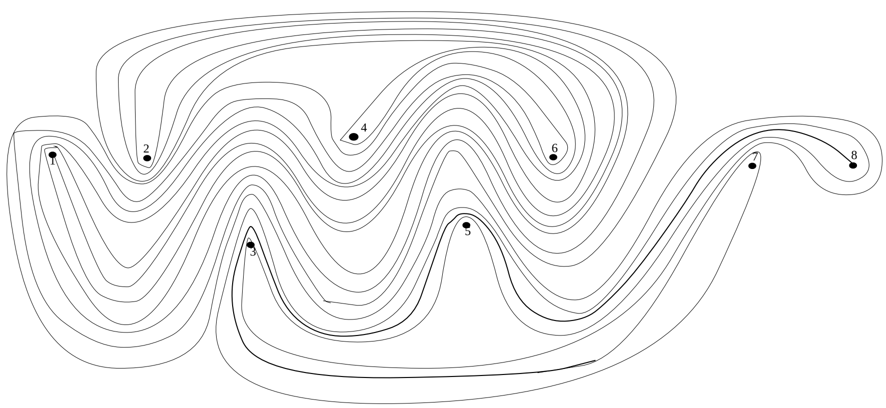
21. Tacnode:



22. Branch point:



23. Branch point:



We compute the group $\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_4)$ in a similar way as above (using the van Kampen Theorem and a long simplification). The starting set of generators for $\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_4)$ is $a_1, \ldots, a_8$. Most of the relations are relatively easy to simplify. The most complicated relations are the ones which are derived from the left branch points (see the braids 18, 20, 22, 23). These relations are translated in fact to relations of the form $(ab)^2 = (ba)^2$. We show now how the branch points relations 18 and 20 are simplified (in the same manner one can also simplify the other ones). Relation 18 is the following one (see the 18th braid):

$$\begin{aligned}&a_1^{-1}a_3^{-1}a_4^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1a_3^{-1}a_5^{-1}a_6a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1\cdot\\&\quad\cdot a_3^{-1}a_5^{-1}a_6a_5a_4a_3\cdot a_2\cdot a_3^{-1}a_4^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_6a_5a_4a_3\cdot\\&\quad\cdot a_2^{-1}a_3^{-1}a_4^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_6a_5a_4a_3a_1\\&=a_3^{-1}a_5^{-1}a_7^{-1}a_8^{-1}a_7a_5\cdot a_3\cdot a_5^{-1}a_7^{-1}a_8a_7a_5a_3.\end{aligned}$$

By relation 2 (see the 2nd braid), we replace $a_3^{-1}a_5^{-1}a_7^{-1}a_8^{-1}a_7a_5a_3a_5^{-1}a_7^{-1}a_8a_7a_5a_3$ with $a_1^{-1}a_2a_1$. So we get:

$$\begin{aligned}&a_1^{-1}a_3^{-1}a_4^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1\cdot\\&\quad\cdot a_3^{-1}a_5^{-1}a_6a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1a_3^{-1}a_5^{-1}a_6a_5a_4a_3\cdot a_2\cdot\\&\quad\cdot a_3^{-1}a_4^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_6a_5a_4a_3a_2^{-1}a_3^{-1}a_4^{-1}a_5^{-1}a_6^{-1}\cdot\\&\quad\cdot a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_6a_5a_4a_3a_1\\&=a_1^{-1}a_2a_1.\end{aligned}$$

Now we use the commutation relation 10 ($[a_3a_2a_3^{-1},a_4]=e$) to simplify the relation to the following form:

$$\begin{aligned}&a_5^{-1}a_6^{-1}a_5a_3a_1a_3^{-1}a_5^{-1}a_6a_5a_3a_2a_3^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1a_3^{-1}a_5^{-1}a_6a_5a_3a_2a_3^{-1}\cdot\\&\quad\cdot a_5^{-1}a_6^{-1}a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_6a_5a_3a_2^{-1}a_3^{-1}a_5^{-1}a_6^{-1}a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_6a_5\\&=a_3a_2a_3^{-1}.\end{aligned}$$

By the commutation relation 11 ($[a_3a_2a_3^{-1},a_5^{-1}a_6a_5]=e$), we simplify the relation to the following form:

$$a_1a_2a_1a_2a_1^{-1}a_2^{-1}a_1^{-1}=a_2,$$

which is $(a_1a_2)^2=(a_2a_1)^2$.

Now let us see how relation 20 can be simplified. Relation 20 is the following one (see the 20th braid):

$$\begin{aligned}&a_3^{-1}a_5^{-1}a_7^{-1}a_8^{-1}a_7a_3^{-1}a_5^{-1}a_7^{-1}a_8a_7a_5a_3a_7a_3^{-1}a_5^{-1}a_7^{-1}a_8a_7a_5a_3\cdot a_7\cdot\\&\quad\cdot a_3^{-1}a_5^{-1}a_7^{-1}a_8^{-1}a_7a_5a_3a_7^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_8^{-1}a_7a_5a_3a_7^{-1}a_8a_7a_5a_3\\&=a_1^{-1}a_3^{-1}a_5^{-1}\cdot a_6\cdot a_5a_3a_1.\end{aligned}$$

By relation 4, we replace $a_1^{-1}a_3^{-1}a_5^{-1}a_6a_5a_3a_1$ with $a_3^{-1}a_5^{-1}a_7^{-1}a_8^{-1}a_7a_8a_7a_5a_3$. So we get:

$$\begin{aligned}&a_7a_3^{-1}a_5^{-1}a_7^{-1}a_8a_7a_5a_3a_7a_3^{-1}a_5^{-1}a_7^{-1}a_8a_7a_5a_3\cdot a_7\cdot\\&\quad\cdot a_3^{-1}a_5^{-1}a_7^{-1}a_8^{-1}a_7a_5a_3a_7^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_8^{-1}a_7a_5a_3a_7^{-1}\\&=a_7.\end{aligned}$$

And now we use relations 6 and 9 ($[a_5, a_7] = e$ and $[a_3, a_7] = e$) to simplify it to the following form:

$$a_8 a_7 a_8 a_7 a_8^{-1} a_7^{-1} a_8^{-1} = a_7,$$

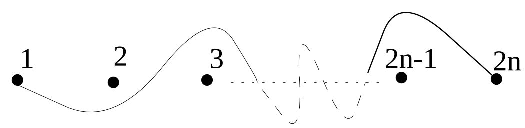
which is $(a_7 a_8)^2 = (a_8 a_7)^2$.

Now, when we know how to deal with the relations, we add the projective relation $a_8 \cdots a_1 = e$ to the presentation, and after some easy simplification we are able to get:

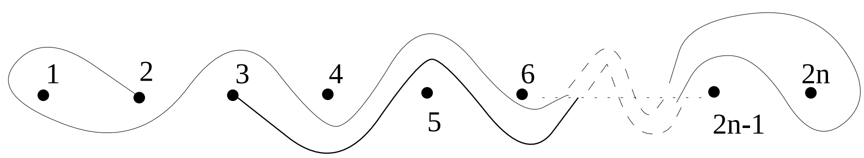
$$\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_4) \simeq \left\langle a_1, a_2, a_3, a_4 \;\middle|\; \begin{array}{l} [a_2, a_3] = [a_2, a_4] = [a_3, a_4] = e \\ (a_1 a_i)^2 = (a_i a_1)^2, \ i = 2, 3, 4 \\ (a_4 a_3 a_2 a_1)^2 = e \end{array} \right\rangle.$$

As for the general case $\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_n)$ (see Figure 9), the argumentation as presented in the arrangements $\mathcal{C}_2, \mathcal{C}_3$ and $\mathcal{C}_4$ actually applies to the general case as well. The initial set of generators for $\pi_1(\mathbb{P}^2 \backslash \mathcal{C}_n)$ is $a_1, \ldots, a_{2n}$. We present now the resulting braids and the induced relations:

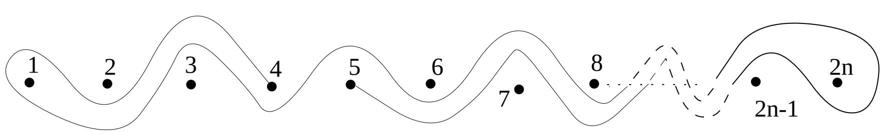
- There are n branch points on the right side of the configuration. They contribute the following braids and relations:



$$a_{2n-1} a_{2n-3} \cdots a_7 a_5 a_3 a_1 a_3^{-1} a_5^{-1} a_7^{-1} \cdots a_{2n-3}^{-1} a_{2n-1}^{-1} = a_{2n},$$

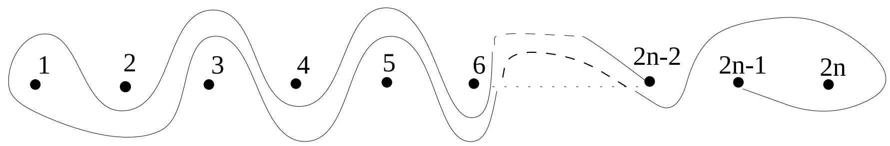


$$a_1^{-1} a_2 a_1 = a_3^{-1} a_5^{-1} a_7^{-1} \cdots a_{2n-3}^{-1} a_{2n-1}^{-1} a_{2n}^{-1} a_{2n-1} a_{2n-3} \cdots a_7 a_5 a_3 a_5^{-1} a_7^{-1} \cdots$$
$$\cdots a_{2n-3}^{-1} a_{2n-1}^{-1} a_{2n} a_{2n-1} a_{2n-3} \cdots a_7 a_5 a_3,$$



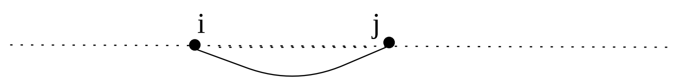
$$a_3 a_1^{-1} a_3^{-1} a_4 a_3 a_1 a_3^{-1} = a_5^{-1} a_7^{-1} \cdots a_{2n-1}^{-1} a_{2n}^{-1} a_{2n-1} \cdots$$
$$\cdots a_7 a_5 a_7^{-1} \cdots a_{2n-1}^{-1} a_{2n} a_{2n-1} \cdots a_7 a_5,$$

and in an inductive way, we get more braids and relations for each two consecutive points, where the last braid is



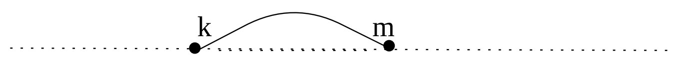
$$
\begin{aligned}
&a_{2n-3}\cdots a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}a_{2n-2}a_{2n-3}\cdots a_5a_3a_1a_3^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}\\
&= a_{2n-1}^{-1}a_{2n}^{-1}a_{2n-1}a_{2n}a_{2n-1}.
\end{aligned}
$$

- The intersection points of the conics contribute three types of braids and relations:



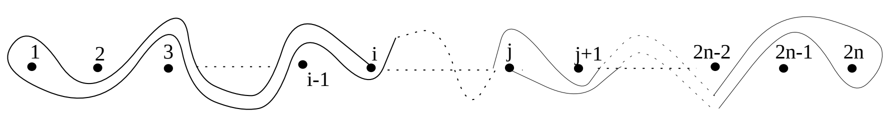
$$[a_i, a_j] = e,$$

for i, j odd and $3 \le i < j \le 2n-1$.



$$[a_{m-1}a_{m-2}\cdots a_{k+1}a_ka_{k+1}^{-1}\cdots a_{m-2}^{-1}a_{m-1}^{-1}, a_m] = e,$$

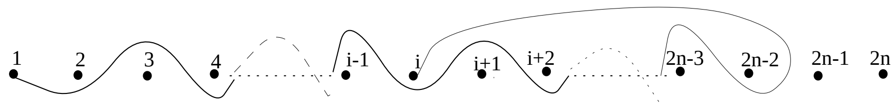
for k, m even and $2 \le k < m \le 2n-2$.



$$
\begin{aligned}
&[a_{i-1}\cdots a_3a_1^{-1}a_3^{-1}\cdots a_{i-1}^{-1}a_ia_{i-1}\cdots a_3a_1a_3^{-1}\cdots a_{i-1}^{-1}, a_{i+1}^{-1}a_{i+3}^{-1}\cdots\\
&\quad \cdots a_{j-2}^{-1}a_j^{-1}a_{j+2}^{-1}\cdots a_{2n-3}^{-1}a_{2n-1}^{-1}a_{2n}^{-1}a_{2n-1}a_{2n-3}\cdots\\
&\quad \cdots a_{j+2}a_ja_{j+2}^{-1}\cdots a_{2n-3}^{-1}a_{2n-1}^{-1}a_{2n}a_{2n-1}a_{2n-3}\cdots a_{j+2}a_ja_{j-2}\cdots a_{i+3}a_{i+1}]\\
&= e,
\end{aligned}
$$

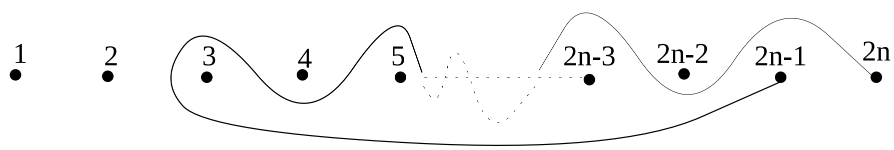
for i even, j odd and $2 \le i \le 2n-2, 3 \le j \le 2n-1$.

- The tacnodes contribute the following braids and relations for i even and $2 \leq i \leq 2n-4$:



$$(a_1 \cdot a_3^{-1} a_5^{-1} \cdots a_{i-1}^{-1} a_{i+1}^{-1} \cdots a_{2n-3}^{-1} a_{2n-2} a_{2n-3} \cdots a_{i+2} a_{i+1} a_i a_{i+1}^{-1} a_{i+2}^{-1} \cdots \\ \cdots a_{2n-3}^{-1} a_{2n-2}^{-1} a_{2n-3} \cdots a_{i+1} a_{i-1} \cdots a_5 a_3)^4 \\ = (a_3^{-1} a_5^{-1} \cdots a_{i-1}^{-1} a_{i+1}^{-1} \cdots a_{2n-3}^{-1} a_{2n-2} a_{2n-3} \cdots a_{i+2} a_{i+1} a_i a_{i+1}^{-1} a_{i+2}^{-1} \cdots \\ \cdots a_{2n-3}^{-1} a_{2n-2}^{-1} a_{2n-3} \cdots a_{i+1} a_{i-1} \cdots a_5 a_3 \cdot a_1)^4,$$

and also



$$(a_{2n} \cdot a_{2n-1} a_{2n-3} \cdots a_5 a_3 a_{2n-1} a_3^{-1} a_5^{-1} \cdots a_{2n-3}^{-1} a_{2n-1}^{-1})^4 \\ = (a_{2n-1} a_{2n-3} \cdots a_5 a_3 a_{2n-1} a_3^{-1} a_5^{-1} \cdots a_{2n-3}^{-1} a_{2n-1}^{-1} \cdot a_{2n})^4.$$

- The branch points on the left side of Figure 9 contribute braids which are being thicker and thicker as long as we proceed to a branch point far away from the typical fiber. The following braid is related to the first branch point (from the right) in this set of branch points (the dotted lines mean: the path is above odd-numbered points and below even-numbered points):

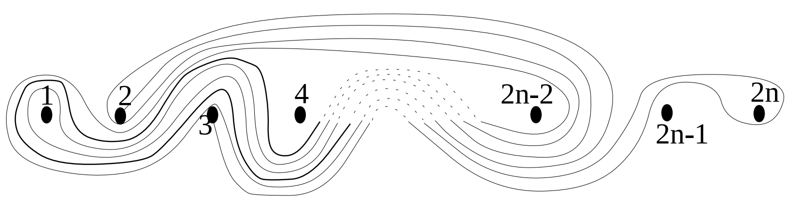
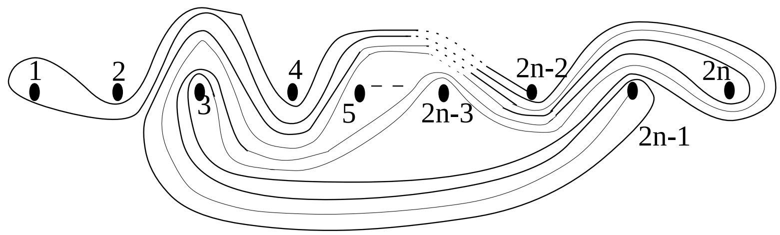


Figure 12

The next braid is related to the second branch point (from the right) in this set of branch points:



and so on.

Here, as in the case of $\mathcal{C}_4$, the relations are relatively easy to simplify, but the ones from the latter set are long and complicated. We show how to simplify the first relation in the latter set (in a similar way we simplify the other ones in this set). This relation is (Figure 12):

$$\begin{aligned}
&a_3^{-1}a_5^{-1}a_7^{-1}\cdots a_{2n-3}^{-1}a_{2n-1}^{-1}a_{2n}^{-1}a_{2n-1}a_{2n-3}\cdots a_7a_5\cdot a_3\cdot a_5^{-1}a_7^{-1}\cdots\\
&\quad\cdots a_{2n-3}^{-1}a_{2n-1}^{-1}a_{2n}a_{2n-1}a_{2n-3}\cdots a_7a_5a_3\\
&= a_1^{-1}a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}a_{2n-2}^{-1}a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots\\
&\quad\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}a_{2n-2}a_{2n-3}\cdots a_5a_4a_3a_2a_3^{-1}\cdot\\
&\quad\cdot a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}a_{2n-2}^{-1}a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots\\
&\quad\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}a_{2n-2}a_{2n-3}\cdots a_5a_4a_3\cdot a_2\cdot a_3^{-1}\cdot\\
&\quad\cdot a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}a_{2n-2}^{-1}a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots\\
&\quad\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}a_{2n-2}a_{2n-3}\cdots a_5a_4a_3a_2^{-1}a_3^{-1}\cdot\\
&\quad\cdot a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}a_{2n-2}^{-1}a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots\\
&\quad\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}a_{2n-2}a_{2n-3}\cdots a_5a_4a_3a_1.
\end{aligned}$$

By the first set of branch points we can substitute $a_1^{-1}a_2a_1$ instead of

$$\begin{aligned}
&a_3^{-1}a_5^{-1}a_7^{-1}\cdots a_{2n-3}^{-1}a_{2n-1}^{-1}a_{2n}^{-1}a_{2n-1}a_{2n-3}\cdots a_7a_5\cdot a_3\cdot a_5^{-1}a_7^{-1}\cdots\\
&\quad\cdots a_{2n-3}^{-1}a_{2n-1}^{-1}a_{2n}a_{2n-1}a_{2n-3}\cdots a_7a_5a_3.
\end{aligned}$$

And by moving $a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}a_{2n-2}^{-1}$ and $a_{2n-2}a_{2n-3}\cdots a_5a_4a_3$ to the other side of the equality, we have now:

$$
\begin{aligned}
&a_{2n-2}(a_{2n-3}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1})a_{2n-2}^{-1}\\
&= a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}\cdot\\
&\quad\cdot a_{2n-2}(a_{2n-3}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1})a_{2n-2}^{-1}\cdot\\
&\quad\cdot a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}\cdot\\
&\quad\cdot a_{2n-2}(a_{2n-3}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1})a_{2n-2}^{-1}\cdot\\
&\quad\cdot a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}\cdot\\
&\quad\cdot a_{2n-2}(a_{2n-3}\cdots a_5a_4a_3a_2^{-1}a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1})a_{2n-2}^{-1}\cdot\\
&\quad\cdot a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}.
\end{aligned}
$$

Now, the commutation relation

$$[a_{m-1}a_{m-2}\cdots a_{k+1}a_ka_{k+1}^{-1}\cdots a_{m-2}^{-1}a_{m-1}^{-1}, a_m] = e,$$

for k, m even and $2 \le k < m \le 2n-2$, can be written as

$$[a_{2n-3}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}, a_{2n-2}] = e,$$

for $k = 2$ and $m = 2n-2$. This relation enables us to omit the element a_{2n-2}:

$$
\begin{aligned}
&a_{2n-3}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1}\\
&= a_{2n-3}a_{2n-5}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}\cdot\\
&\quad\cdot a_{2n-3}^{-1}(a_{2n-3}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1})a_{2n-3}\cdot\\
&\quad\cdot a_{2n-5}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}\cdot\\
&\quad\cdot a_{2n-3}^{-1}(a_{2n-3}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1})a_{2n-3}\cdot\\
&\quad\cdot a_{2n-5}\cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}\cdot\\
&\quad\cdot a_{2n-3}^{-1}(a_{2n-3}\cdots a_5a_4a_3a_2^{-1}a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-3}^{-1})a_{2n-3}\cdot\\
&\quad\cdot a_{2n-5}\cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}a_{2n-3}^{-1}.
\end{aligned}
$$

At this stage, the element a_{2n-3} can be omitted too, so we have now:

$$
\begin{aligned}
&a_{2n-4}(a_{2n-5}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-5}^{-1})a_{2n-4}^{-1}\\
&= a_{2n-5}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}\cdot\\
&\quad\cdot a_{2n-4}(a_{2n-5}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-5}^{-1})a_{2n-4}^{-1}\cdot\\
&\quad\cdot a_{2n-5}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}\cdot\\
&\quad\cdot a_{2n-4}(a_{2n-5}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-5}^{-1})a_{2n-4}^{-1}\cdot\\
&\quad\cdot a_{2n-5}\cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}\cdot\\
&\quad\cdot a_{2n-4}(a_{2n-5}\cdots a_5a_4a_3a_2^{-1}a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-5}^{-1})a_{2n-4}^{-1}\cdot\\
&\quad\cdot a_{2n-5}\cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-5}^{-1}.
\end{aligned}
$$

We use again the above commutation relation

$$[a_{2n-5}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-5}^{-1}, a_{2n-4}] = e,$$

for $k=2$ and $m=2n-4$, to omit a_{2n-4}. Omitting this element, enables us to omit also a_{2n-5}:

$$\begin{aligned}
&a_{2n-6}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-6}^{-1}\\
&= a_{2n-7}\cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots\\
&\quad \cdots a_{2n-7}^{-1}(a_{2n-6}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-6}^{-1})a_{2n-7}\cdots\\
&\quad \cdots a_9a_7a_5a_3a_1a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots\\
&\quad \cdots a_{2n-7}^{-1}(a_{2n-6}\cdots a_5a_4a_3a_2a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-6}^{-1})a_{2n-7}\cdots\\
&\quad \cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots\\
&\quad \cdots a_{2n-7}^{-1}(a_{2n-6}\cdots a_5a_4a_3a_2^{-1}a_3^{-1}a_4^{-1}a_5^{-1}\cdots a_{2n-6}^{-1})a_{2n-7}\cdot\\
&\quad \cdots a_9a_7a_5a_3a_1^{-1}a_3^{-1}a_5^{-1}a_7^{-1}a_9^{-1}\cdots a_{2n-7}^{-1}.
\end{aligned}$$

We continue in the same way to omit elements till we get the following form:

$$\begin{aligned}
a_4a_3a_2a_3^{-1}a_4^{-1} = {}& a_3a_1a_3^{-1}a_4a_3a_2a_3^{-1}a_4^{-1}a_3a_1a_3^{-1}a_4a_3a_2a_3^{-1}a_4^{-1}a_3a_1^{-1}a_3^{-1}a_4\cdot\\
&\cdot a_3a_2^{-1}a_3^{-1}a_4^{-1}a_3a_1^{-1}a_3^{-1}.
\end{aligned}$$

And in the last time we use the same commutation relation in the form of

$$[a_3a_2a_3^{-1}, a_4] = e,$$

for $k=2, m=4$, to get

$$a_2 = a_1a_2a_1a_2a_1^{-1}a_2^{-1}a_1^{-1},$$

which is $(a_1a_2)^2 = (a_2a_1)^2$.

After simplifying the latter set of relations and the other sets of relations (which are easy to simplify), we add the projective relation $a_{2n}a_{2n-1}\cdots a_2a_1 = e$. This enables us to continue the simplification and to get the presentation for $\pi_1(\mathbb{P}^2\backslash\mathcal{C}_n)$, as it appears in (23) - this presentation replaces the one from the paper, Theorem 2.8, page 265. $\square$

We add a corollary which does not appear in the original paper, but holds.

Corollary 1.2. *The group $\pi_1(\mathbb{P}^2\backslash\mathcal{C}_n)$ is abelian for $n=1$ and big for $n\geq 2$.*

Proof. If we forget the indices $i\geq 3$ in (23), we get $\pi_1(\mathbb{P}^2\backslash\mathcal{A}_2)$ which is isomorphic to $\mathbb{Z}*\mathbb{Z}_2$, and from this point, it clear that $\pi_1(\mathbb{P}^2\backslash\mathcal{C}_n)$ is big (see the proof of New Corollary 2.4). $\square$

2. Wrong stated results in the paper

Proposition 4.1 (page 270 in the paper) and its Corollary 4.2 (page 271 in the paper) need to be rephrased: as the proofs in the paper show, the monodromy is a quadruple fulltwist, not a double fulltwist as erroneously stated.

New Proposition 4.1. *Let O be a unique tacnode between n quadrics. Then the local monodromy around O is a quadruple fulltwist on the disk.*

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