

On a reaction-diffusion system involving the critical exponent.

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Abstract

In this paper we study the existence and multiplicity of the nontrivial solutions for the following elliptic system with Dirichlet boundary conditions and critical nonlinearity

$$\begin{cases} -\Delta u = \lambda u + W(x)u|u|^{2^*-2} - kv & \text{in } \Omega \\ -\Delta v = \delta u - \gamma v & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases},$$

where $\Omega \subset \mathbf{R}^N (N \geq 3)$ is a bounded regular domain, $W(\cdot) \in L^\infty(\Omega)$ with the property that there exists $\eta > 0$ such that $W(\cdot) \geq \eta$ a.e. in Ω and λ, δ, γ are real parameters. We show that the number of nontrivial solutions, in a left neighbourhood of each $\widehat{\lambda}_j, j = 1, 2, \dots$, is at least twice the multiplicity of $\widehat{\lambda}_j$, where the set $\{\widehat{\lambda}_j\}_{j \in \mathbf{N}^*}$ represents the spectrum of a certain integro-differential operator.

1 Introduction

Rothe in [R] considered the system of reaction diffusion equations

$$\begin{cases} \partial u \partial t = \mu \Delta u + f(u) - v \\ \varepsilon \partial v \partial t = \Delta v + u - v \end{cases}, \quad (1)$$

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for $(t, x) \in (0, \infty) \times \Omega$. Here u, v are real functions of $(t, x) \in [0, \infty) \times \overline{\Omega}$, where $\Omega \subset \mathbf{R}^N$ ($N \geq 1$) is open, bounded and connected. As explained in [RM], u and v , which are called the activator and inhibitor respectively, can be interpreted as relative concentrations of substances known as morphogens. The system (1) is supplemented by Dirichlet boundary conditions

$$u = v = 0, \text{ for } (t, x) \in (0, \infty) \times \partial\Omega$$

and the initial conditions

$$u(0, x) = u_0(x), v(0, x) = v_0(x), \text{ for all } x.$$

As shown in [RM], the existence of equilibrium solutions in (1) is determined by the problem with $\varepsilon = 0$ and the equilibrium states are solutions of the elliptic system

$$\begin{cases} \mu \Delta u + f(u) - v = 0 & \text{in } \Omega \\ \Delta v + u - v = 0 & \text{in } \Omega \end{cases}$$

subject to Dirichlet boundary conditions

$$u = v = 0 \text{ on } \partial\Omega.$$

It will be convenient to split the function f , which models autocatalytic and saturation effects, into the linear and higher order terms

$$f(u) = \lambda u + g(u).$$

Notation. In the rest of the paper we make use of the following notation

$L^p(\Omega)$, $1 \leq p \leq \infty$, denote Lebesgue spaces; the norm in L^p is denoted by $\|\cdot\|_p$;

$W^{k,p}(\Omega)$ denote Sobolev spaces;

$H_0^1(\Omega)$ denotes $W_0^{1,2}(\Omega)$, endowed with the norm $\|u\|^2 = \int_{\Omega} |\nabla u|^2 dx$;

$H^{-1}(\Omega)$ denotes the topological dual of $H_0^1(\Omega)$; the norm in this space is denoted by $\|\cdot\|_{H^{-1}}$.

We consider below the problem of finding nontrivial solutions of the slightly more general elliptic system with Dirichlet boundary conditions and critical nonlinearity

$$(P) \quad \begin{cases} -\Delta u = \lambda u + W(x)u|u|^{2^*-2} - kv & \text{in } \Omega \\ -\Delta v = \delta u - \gamma v & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases},$$

where $\Omega \subset \mathbf{R}^N (N \geq 3)$ is a bounded regular domain, δ, γ and k are constants such that $k\delta > 0$ and $\gamma > -\lambda_1(\Omega)$, where $\lambda_1(\Omega)$ is the first eigenvalue of the Dirichlet Laplacian on Ω , and $W(\cdot) \in L^\infty(\Omega)$ with the property that there exists $\eta > 0$ such that $W(\cdot) \geq \eta$ a.e. in Ω . Here $2^* = 2N / (N - 2)$.

In the subcritical case the system (1) has been studied by various authors (see [Ro], [Si], [FM], [NT] and others). The review, even partial, of their results is out of the scope of this paper.

Assuming u to be known, the Dirichlet boundary value problem

$$\begin{cases} -\Delta v + \gamma v = \delta u & \text{in } \Omega \\ v = 0 & \text{on } \partial\Omega \end{cases}$$

is uniquely solved by $v = 1/k B u$ where the operator $B = k\delta (-\Delta + \gamma)^{-1}$ is bounded from $L^p(\Omega)$ to $W^{2,p}(\Omega)$ for all $1 \leq p < \infty$. Also, by the Schauder theory, B maps the Hölder space $C^\alpha(\bar{\Omega})$ into $C^{1+\alpha}(\bar{\Omega})$.

Moreover, it is easily checked that B is positive and self-adjoint in the sense that

$$\int_{\Omega} u B u dx = \frac{1}{k\delta} \int_{\Omega} |\nabla w|^2 + \gamma w^2 dx$$

for $u \in L^2(\Omega)$ and $w = B u$; and if $w = B u, z = B v$ then

$$\int_{\Omega} u B v dx = \frac{1}{k\delta} \int_{\Omega} \nabla w \nabla z + \gamma w z dx = \int_{\Omega} v B u dx.$$

Let us define the operator

$$T \equiv -\Delta + B : L^2(\Omega) \rightarrow L^2(\Omega), \text{ with } D(T) = W^{2,2}(\Omega) \cap H_0^1(\Omega).$$

It is easy to observe that T is symmetric on its domain $D(T)$ i.e.

$$\langle T u_1, u_2 \rangle = \langle u_1, T u_2 \rangle \text{ for all } u_1, u_2 \in D(T),$$

where $\langle \cdot, \cdot \rangle$ denotes the L^2 -inner product.

If $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \dots$ and $(\varphi_k)_k$ denote respectively the eigenvalues and the eigenfunctions of $-\Delta$ in Ω under zero Dirichlet boundary conditions, then one can verify easily that the φ_k 's are also eigenfunctions of T corresponding to the modified eigenvalues

$$\widehat{\lambda}_k = \lambda_k + \frac{k\delta}{\gamma + \lambda_k}, \quad k = 1, 2, \dots$$

A more detailed analysis shows that the spectrum $\sigma(T)$ of T consists precisely of these eigenvalues (see [FM, Corollary 1.2.]).

From the above, we obtain that (P) is equivalent to the integro-differential equation

$$(P') \quad \begin{cases} -\Delta u + Bu = \lambda u + W(x)u|u|^{2^*-2} & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

We associate to the problem (P') the functional

$$I_\lambda(u) = \frac{1}{2} \int_{\Omega} |\nabla u|^2 + uBu - \lambda u^2 dx - \frac{1}{2^*} \int_{\Omega} W(x) |u|^{2^*} dx, \quad \forall u \in H_0^1(\Omega).$$

In a standard way we can prove that $I_\lambda \in C^1(H_0^1(\Omega), \mathbf{R})$ and the critical points of I_λ are solutions of (P').

Note that $p = 2^*$ is the limiting Sobolev exponent for the embedding $H_0^1(\Omega) \hookrightarrow L^p(\Omega)$. Since this embedding is not compact, the functional I_λ does not satisfy the Palais-Smale condition in the energy range $(-\infty, +\infty)$. Hence there are serious difficulties when trying to find critical points by standard variational methods.

Using the ideas of Pohozaev (see [P]), Figueiredo and Mitidieri obtained a similar identity for the system (P) (see [FM, Lemma 4.1 and Remark 2.7]). From this identity, if Ω is starshaped, we can obtain that (P) admits only the trivial solution $u \equiv v \equiv 0$ for $\lambda \leq 0$.

Denote

$$S_B = \inf_{u \in H_0^1(\Omega) \setminus \{0\}} \frac{\|u\|_B^2}{\|u\|_{2^*}^2}$$

where $\|u\|_B^2 = \int_{\Omega} |\nabla u|^2 + uB u dx$, $\forall u \in H_0^1(\Omega)$. From the positivity of B we have that

$$S_B \geq S = \inf_{u \in H_0^1(\Omega) \setminus \{0\}} \frac{\|u\|^2}{\|u\|_{2^*}^2},$$

where S corresponds to the best constant for the Sobolev continuous embedding $H_0^1(\Omega) \hookrightarrow L^{2^*}(\Omega)$. Then $S_B > 0$ because it is well known that $S > 0$.

Under the above conditions and notations, the result proved in this paper is the following:

Theorem 1.1. For $\lambda > 0$ denote $\widehat{\lambda}_+ = \min \{ \widehat{\lambda}_j : \lambda < \widehat{\lambda}_j \}$ and suppose that the multiplicity of $\widehat{\lambda}_+$ is m . Then, if

$$\widehat{\lambda}_+ - \lambda < \left(\frac{\eta}{\|W\|_\infty} \right)^{\frac{2}{2^*}} S_B [\text{meas}(\Omega)]^{-2/N},$$

the problem (P) admits at least m pairs of nontrivial solutions

$$\{(u_k(\lambda), v_k(\lambda)); (-u_k(\lambda), -v_k(\lambda))\}, k = 1, 2, \dots, m.$$

Moreover

$$\|u_k(\lambda)\| \rightarrow 0 \text{ and } \|v_k(\lambda)\| \rightarrow 0, \text{ as } \lambda \nearrow \widehat{\lambda}_+,$$

for every

$$k \in \{1, 2, \dots, m\}.$$

The proof of the above theorem uses standard ideas and the techniques are essentially the same as those used in [CFS] and [CFP]. The main tool used is the following slightly modified result of Bartolo, Benci and Fortunato (see [BBF, Theorem 2.4]) contained in [CFS, Theorem 2.5]:

Theorem 2.2. Let H be a real Hilbert space with norm $\|\cdot\|_H$ and suppose $I \in C^1(H, \mathbf{R})$ is a functional on H satisfying the following conditions:

I1) I is even, $I(0) = 0$;

I2) There exists a constant $\beta > 0$ such that the Palais-Smale condition (PS) holds in $(0, \beta)$;

I3) There exist two closed subspaces $V, W \subset H$ and positive constants ρ, ξ, β' with $\xi < \beta' < \beta$ such that

i) $I(u) \leq \beta'$ for any $u \in W$;

ii) $I(u) \geq \xi$ for any $u \in V, \|u\|_H = \rho$;

iii) $\text{codim}V < \infty$ and $\text{dim}W \geq \text{codim}V$.

Then there exists at least $\text{dim}W - \text{codim}V$ pairs of critical points of I with critical values belonging to the interval $[\xi, \beta']$.

2 Proof of Theorem 1

Step1.

First we show that although the Palais-Smale condition does not hold globally for I_λ it is satisfied locally in $(-\infty, 1 N S_B^{N/2} \|W\|_\infty^{\frac{N-2}{2}})$ in the following sense:

If $c < 1 N S_B^{N/2} \|W\|_\infty^{\frac{N-2}{2}}$ and $(u_m)_{m \geq 1}$ is a sequence in $H_0^1(\Omega)$ such that

$$\begin{cases} I_\lambda(u_m) \rightarrow c \\ dI_\lambda(u_m) \rightarrow 0 \text{ strongly in } H^{-1}(\Omega) \end{cases}, \text{ as } m \rightarrow \infty,$$

then $(u_m)_{m \geq 1}$ contains a subsequence converging strongly in $H_0^1(\Omega)$.

Let $c \in (-\infty, 1 N S_B^{N/2} \|W\|_\infty^{\frac{N-2}{2}})$ and let $(u_m)_{m \geq 1} \subset H_0^1(\Omega)$ be a sequence such that

$$\begin{aligned} I_\lambda(u_m) &\rightarrow c, \text{ as } m \rightarrow \infty, \text{ and} \\ dI_\lambda(u_m) &\rightarrow 0, \text{ as } m \rightarrow \infty, \text{ in } H^{-1}(\Omega). \end{aligned}$$

It is easy to observe that there exists $M > 0$ a positive constant such that, for every $m \in \mathbb{N}^*$, $|I_\lambda(u_m)| \leq M$.

If we choose $\theta \in (12^*, 12)$ and $m \in \mathbb{N}^*$ sufficiently large, we obtain

$$\begin{aligned} M + \theta \|u_m\| &\geq I_\lambda(u_m) - \theta d I_\lambda(u_m) u_m \geq \frac{1}{2} \int_\Omega |\nabla u_m|^2 + u_m B u_m - \lambda u_m^2 dx - \\ &- \frac{1}{2^*} \int_\Omega W(x) |u_m|^{2^*} dx - \theta \int_\Omega |\nabla u_m|^2 + u_m B u_m - \lambda u_m^2 dx + \theta \int_\Omega W(x) |u_m|^{2^*} dx \geq \\ &\geq \left(\frac{1}{2} - \theta\right) \int_\Omega |\nabla u_m|^2 + u_m B u_m - \lambda u_m^2 dx + \left(\theta - \frac{1}{2^*}\right) \int_\Omega W(x) |u_m|^{2^*} dx \geq \\ &\geq \left(\frac{1}{2} - \theta\right) \|u_m\|^2 - C_1 \lambda \|u_m\|_{2^*}^2 + \eta \left(\theta - \frac{1}{2^*}\right) \|u_m\|_{2^*}^{2^*} \geq \\ &\geq \left(\frac{1}{2} - \theta\right) \|u_m\|^2 + \inf_{\rho \geq 0} \left[\eta \left(\theta - \frac{1}{2^*}\right) \rho^{2^*} - C_1 \lambda \rho^2 \right], \end{aligned}$$

where $C_1 > 0$ is a positive constant.

Then $(u_m)_{m \geq 1}$ is bounded in $H_0^1(\Omega)$. Hence we may extract a subsequence $(u_m)_{m \geq 1}$ (relabelled) such that

$$\begin{aligned} u_m &\rightharpoonup u \text{ weakly in } H_0^1(\Omega) \\ u_m &\rightarrow u \text{ strongly in } L^p(\Omega), \text{ for any } p \in [1, 2^*) \\ u_m &\rightarrow u \text{ a.e. in } \Omega \end{aligned}$$

Now, we prove that u is a solution of (P') . Let $\varphi \in C_0^\infty(\Omega)$. Then

$$|dI_\lambda(u) \varphi| \leq \|dI_\lambda(u_m)\|_{H^{-1}} \|\varphi\| + |(dI_\lambda(u) - dI_\lambda(u_m)) \varphi| \rightarrow 0, \text{ as } m \rightarrow \infty.$$

Hence u weakly solves (P') .

Let $v_m = u_m - u$. Clearly

$$v_m \rightharpoonup 0 \text{ weakly in } H_0^1(\Omega) \tag{2}$$

$$v_m \rightarrow 0 \text{ strongly in } L^p(\Omega), \text{ for any } p \in [1, 2^*) \tag{3}$$

$$v_m \rightarrow 0 \text{ a.e. in } \Omega$$

From (2) and (3) observe that

$$\begin{aligned} o(1) &= dI_\lambda(u_m) v_m = \int_\Omega \nabla u_m \nabla v_m + v_m B u_m - \lambda u_m v_m dx - \\ &\quad - \int_\Omega W(x) v_m u_m |u_m|^{2^*-2} dx \\ &= \int_\Omega |\nabla v_m|^2 + v_m B v_m dx - \int_\Omega W(x) v_m u_m |u_m|^{2^*-2} dx + o(1) \\ &= \|v_m\|_B^2 - \int_\Omega W(x) v_m u_m |u_m|^{2^*-2} dx + o(1). \end{aligned}$$

Hence

$$\|v_m\|_B^2 = \int_\Omega W(x) v_m u_m |u_m|^{2^*-2} dx + o(1) \leq \|W\|_\infty \int_\Omega |v_m|^{2^*} dx + o(1). \tag{4}$$

Since

$$dI_\lambda(u_m) u_m = o(1),$$

we have that

$$\int_{\Omega} W(x) |u_m|^{2^*} dx = \int_{\Omega} |\nabla u_m|^2 + u_m B u_m - \lambda u_m^2 dx + o(1).$$

Using this last equality we obtain

$$\begin{aligned} I_{\lambda}(u_m) &= \frac{1}{2} \left(\|u_m\|_B^2 - \lambda \|u_m\|_2^2 \right) - \frac{1}{2^*} \int_{\Omega} W(x) |u_m|^{2^*} dx \geq \\ &\geq \frac{\eta}{N} \|u\|_{2^*}^2 + \frac{1}{N} \|v_m\|_B^2 + o(1) \geq \frac{1}{N} \|v_m\|_B^2 + o(1). \end{aligned}$$

Then

$$\|v_m\|_B^2 \leq N I_{\lambda}(u_m) + o(1) < S_B^{N/2} \|W\|_{\infty}^{\frac{N-2}{2}}, \text{ for } m \text{ sufficiently large.} \quad (5)$$

From (4) we have

$$\begin{aligned} \|v_m\|_B^2 &\leq \|W\|_{\infty} S_B^{-\frac{2^*}{2}} \|v_m\|_B^{2^*} + o(1) \iff \\ \|v_m\|_B^2 &\left(S_B^{\frac{2^*}{2}} - \|W\|_{\infty} \|v_m\|_B^{2^*-2} \right) \leq o(1). \end{aligned}$$

Since, from (5),

$$S_B^{\frac{2^*}{2}} > \|W\|_{\infty} \|v_m\|_B^{2^*-2} \text{ for } m \text{ large enough,}$$

we obtain that

$$v_m \rightarrow 0, \text{ strongly in } H_0^1(\Omega), \text{ as } m \rightarrow \infty,$$

and this ends the proof of the fact that I_{λ} satisfies the Palais-Smale condition on $\left(-\infty, 1 N S_B^{N/2} \|W\|_{\infty}^{\frac{N-2}{2}}\right)$.

Step 2.

Set

$$H_1 = \overline{\widehat{\lambda}_j \geq \widehat{\lambda}_+ \oplus M(\widehat{\lambda}_j)} \text{ and } H_2 = \widehat{\lambda}_j \leq \widehat{\lambda}_+ \oplus M(\widehat{\lambda}_j),$$

where $M(\widehat{\lambda}_j)$ denotes the eigenspace of T corresponding to the eigenvalue $\widehat{\lambda}_j$. Denote $\beta_\lambda = H_2 \sup I_\lambda$ and observe that, if $u = \sum_{\widehat{\lambda}_i \leq \widehat{\lambda}_+} a_i \varphi_i \in H_2$, we have

$$\begin{aligned} I_\lambda(u) &= \frac{1}{2} \|u\|_B^2 - \lambda \|u\|_2^2 - \frac{1}{2^*} \int_\Omega W(x) |u|^{2^*} dx \leq \frac{1}{2} (\widehat{\lambda}_+ - \lambda) \\ &\quad \int_\Omega u^2 dx - \frac{\eta}{2^*} \|u\|_{2^*}^{2^*} \leq \frac{1}{2} (\widehat{\lambda}_+ - \lambda) (\text{meas } \Omega)^{2/N} \|u\|_{2^*}^2 - \frac{\eta}{2^*} \|u\|_{2^*}^{2^*} \\ &\leq \rho \geq 0 \sup \left[\frac{1}{2} (\widehat{\lambda}_+ - \lambda) (\text{meas } \Omega)^{2/N} \rho^2 - \frac{\eta}{2^*} \rho^{2^*} \right] \\ &= \frac{1}{N} \eta^{\frac{2-N}{2}} (\widehat{\lambda}_+ - \lambda)^{N/2} (\text{meas } \Omega). \end{aligned}$$

Thus

$$\beta_\lambda \leq \frac{1}{N} \eta^{\frac{2-N}{2}} (\widehat{\lambda}_+ - \lambda)^{N/2} (\text{meas } \Omega).$$

If $u = \sum_{\widehat{\lambda}_i \geq \widehat{\lambda}_+} a_i \varphi_i \in H_1$, a simple computation shows that

$$I_\lambda(u) \geq \left(1 - \frac{\lambda}{\widehat{\lambda}_+}\right) \|u\|_B^2 - C_2 \|u\|_B^{2^*},$$

where $C_2 > 0$ is a positive constant. Clearly, there exist constants $\rho_\lambda, \xi_\lambda \in (0, \beta_\lambda)$ such that

$$I_\lambda(u) \geq \xi_\lambda, \text{ for any } u \in H_1, \|u\|_B = \rho_\lambda.$$

Step 3.

Now, it is easy to observe that the hypothesis of Theorem 2 are satisfied for $H = H_0^1(\Omega)$, $f = I_\lambda$, $\beta = 1 N S_B^{N/2} \|W\|_\infty^{\frac{N-2}{2}}$, $V = H_1$, $W = H_2$, $\xi = \xi_\lambda$, $\rho = \rho_\lambda$, $\beta' = \beta_\lambda$ and so, for

$$\widehat{\lambda}_+ - \lambda < \left(\frac{\eta}{\|W\|_\infty} \right)^{\frac{2}{2^*}} S_B [\text{meas } \Omega]^{-2/N},$$

the problem (P') admits at least

$$m = \dim (H_1 \cap H_2) - \text{codim} (H_1 + H_2) = \dim M (\widehat{\lambda}_+)$$

pairs of nontrivial solutions

$$\{u_k(\lambda), -u_k(\lambda)\}, k = 1, 2, \dots, m.$$

Since

$$I_\lambda(u_k(\lambda)) \in [\delta, \beta'] \text{ and } \beta' \leq \frac{1}{N} \eta^{\frac{2-N}{2}} (\widehat{\lambda}_+ - \lambda)^{N/2} (\text{meas}(\Omega)) \rightarrow 0, \text{ as } \lambda \nearrow \widehat{\lambda}_+,$$

we obtain that

$$I_\lambda(u_k(\lambda)) \rightarrow 0, \text{ as } \lambda \nearrow \widehat{\lambda}_+, \forall k \in \{1, 2, \dots, m\}.$$

From this and from $dI_\lambda(u_k(\lambda)) = 0$, we obtain that

$$u_k(\lambda) \rightarrow 0, \text{ strongly in } H_0^1(\Omega), \text{ as } \lambda \nearrow \widehat{\lambda}_+. \quad (6)$$

since I_λ satisfies the (PS) condition in the interval

$$\left(-\infty, 1 N S_B^{N/2} \|W\|_\infty^{\frac{N-2}{2}}\right).$$

Now, from the equivalence between (P') and (P) , it is easy to observe that if $\widehat{\lambda}_+ - \lambda < (\eta \|W\|_\infty)^{\frac{2}{N}} S_B [\text{meas}(\Omega)]^{-2/N}$, then (P) admits at least m pairs of nontrivial solutions $\{(u_k(\lambda), v_k(\lambda)); (-u_k(\lambda), -v_k(\lambda))\}$, $k = 1, 2, \dots, m$, where $v_k(\lambda) = 1 k B(u_k(\lambda))$. Moreover, from (6) and the continuity of B , we also obtain that

$$v_k(\lambda) \rightarrow 0, \text{ strongly in } H_0^1(\Omega), \text{ as } \lambda \nearrow \widehat{\lambda}_+.$$

and this ends the proof.

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