ILL-POSED PROBLEMS

Rome, November 29 - 30, 2010

Program

Monday, November 29th, 2010, morning

10.30 – 11.15

speaker Jesùs Ildefonso DIAZ (Universidad Complutense de Madrid, Spain)

On time periodic free boundaries in distributed systems: The last paper by Maurizio Badii.

11.15 – 11.45 **Coffee break**

11.45 – 12.30

speaker Michel CHIPOT (Universität Zürich, Switzerland)

On some exotic problems.

12.30 – 13.15

speaker Angel Manuel RAMOS (Universidad Complutense de Madrid, Spain)

Modelling and simulation of high pressure processes in food engineering and corresponding inverse problems.

Monday, November 29th, 2010, afternoon

13.15 – 14.30 **Lunch**

14.30 – 15.15

speaker Danielle HILHORST (Université Paris - Sud, Orsay, France)

A nonlinear cross-diffusion system for contact inhibition of cell growth.

14.30 – 15.15

speaker Laurent COHEN (Université de Paris - Dauphine, France)

Extraction of tubular and tree structures in biomedical images using minimal paths and tubular models.

16.30 – 18.30

Meeting of the Governing Board.

Tuesday, November 30th, 2010, morning

10.00 – 10.45

speaker Günther GRÜN (Universität Erlangen - Nürnberg, Germany)

Electrowetting with electrolytes:
Existence for an elliptic-parabolic system with critical coupling.

10.45 – 11.30

speaker Peter KNABNER (Universität Erlangen - Nürnberg, Germany)

Hierarchical and adaptive concept for the identification of material laws by boundary observations.

11.30 – 12.00 **Coffee break**

12.00 – 12.45

speaker Lorenzo GIACOMELLI ("Sapienza" Università di Roma, Italy)

A dissipative system from strain-gradient plasticity.

12.45 – 13.30

speaker Flavia SMARRAZZO ("Sapienza" Università di Roma, Italy)

Large-time behaviour of weak entropy measure-valued solutions to a class of equations with variable parabolicity direction.

Tuesday, November 30th, 2010, afternoon

13.30 – 14.45 **Lunch**

14.45 – 15.30

speaker Nicholas ALIKAKOS (University of Athens, Greece)

Heteroclinic traveling waves of gradient diffusion systems.

15.30 – 16.15

speaker Johannes ZIMMER (University of Bath, United Kingdom)

Ill-posed problem in nonlinear elasticity: Macro and micro.

16.15 – 16.45 **Coffee break**

16.45 – 17.30

speaker Giovanni BELLETTINI ("Tor Vergata" Università di Roma, Italy)

Remarks on a Perona-Malik type equation:
Convergence of discrete approximations for large times.

Titles, abstracts & schedule

Nicholas D. Alikakos

University of Athens, Greece

title **Heteroclinic traveling waves of gradient diffusion systems**.

abstract We establish existence of a traveling wave to a parabolic gradient system connecting two minima of the potential.

This is joint work with N. Katzourakis. It is based on previous joint work with G. Fusco.

14.45, Tuesday, November 30, 2010.

Giovanni Bellettini

"Tor Vergata" Università di Roma, Italy

title **Remarks on a Perona-Malik type equation: Convergence of discrete approximations for large times**.

abstract Using an asymptotic expansion argument, we discuss a rigorous convergence result of a semidiscrete Perona-Malik type equation as the grid size goes to zero, in a suitable large time scale. Despite the fact that the original equation is forward-backward, a comparison argument based on the construction of suitable sub/supersolutions guarantees the convergence to a limit system of ordinary differential equations.

16.45, Tuesday, November 30, 2010.

Michel Chipot

Universität Zürich, Switzerland

title **On some exotic problems**.

abstract We would like to consider problems which in a simple form could be written as

$$-a\left(\int_\Omega u dx\right)\partial^2_{x_2}u = f(x_1,x_2) \quad \text{in } \Omega := (0,1)^2$$

together with boundary conditions.

Joint work with S. Guesmia.

11.45, Monday, November 29, 2010.

Laurent Cohen

Université de Paris - Dauphine, France

title **Extraction of tubular and tree structures in biomedical images using minimal paths and tubular models**.

abstract Tubular and tree structures appear very commonly in biomedical images like vessels, microtubules or in different imaging of neuron cells. Minimal paths have been used for long in order to segment these structures. They are a way to find a (set of) curve(s) globally minimizing the geodesic active contours energy. It can be solved by the Eikonal equation using the fast and efficient Fast Marching method. The user usually provides start and end points and gets the minimal path as output.

In the past years we have introduced different extensions of these minimal paths that improve either the interactive aspects or the results. For example, we proposed a way to obtain a closed curve from a single initial point by adding iteratively what we called the keypoints. The result is then a set of minimal paths between pairs of keypoints. This can also be applied to branching structures in both 2D and 3D images. We also proposed different criteria to obtain automatically a set of end points of a tree structures by giving only one starting point. More recently, we proposed a method that takes into account both scale and orientation of the path. This leads to solving an anisotropic minimal path in a 2D or 3D+radius space.

he work we will present involved as well F. Benmansour, Y. Rouchdy and J. Mille at CEREMADE.

15.15, Monday, November 29, 2010.

Jesús Ildefonso Diaz

Universidad Complutense de Madrid, Spain

title **On time periodic free boundaries in distributed systems: The last paper by Maurizio Badii**.

abstract Very often, many natural and social phenomena are modeled in terms of nonlinear parabolic partial differential equations with time periodical data (source forcing terms and/or boundary conditions). The main goal of this talk is to present some sufficient conditions, being also necessary in many cases, for the existence of a periodic free boundary generated as the boundary of the support of the periodic solution of a general class of nonlinear parabolic equations.
We show some qualitative properties of this free boundary. In some cases it may have some vertical shape (linking the free boundaries associated to two different stationary solutions) and, under the assumption of a strong absorption, it may have several periodic connected components. Most of the results applies to some

ill-posed problems, since they are established under a great generality which does not require a complete information on the equation neither on the type of boundary condition (for instance, it is enough to know some L^∞-local estimates outside the support of the data). Some of the results can be proven by energy methods and so they also holds for suitable higher order equations of Cahn-Hilliard type.

The results, in collaboration with Maurizio Badii, were obtained only some months before his death (November 2, 2009). The best which could be found in this lecture is dedicated to his memory.

10:30, Monday, November 29, 2010.

Lorenzo Giacomelli

"Sapienza" Università di Roma, Italy

title **A dissipative system from strain-gradient plasticity**.

abstract I will discuss a nonlocal and nonlinear (possibly rate-independent) system of PDEs related to a plasticity theory proposed by M. Gurtin. Such theories are expected to describe the strong dependence on the size of the specimen which metals display, over a scale which extends from about a fraction of a micron to tens of microns, when deformed non uniformly into the plastic range: smaller specimens appear to have higher relative strength. After reviewing recent achievements concerning well-posedness, I will report on a few ongoing projects which aim to obtain qualitative and quantitative information on the scaling properties of solutions by suitable simplifications of the system.

12.00, Tuesday, November 30, 2010.

Günther Grün

Universität Erlangen - Nürnberg, Germany

title **Electrowetting with electrolytes:**
Existence for an elliptic-parabolic system with critical coupling.

abstract We prove existence of weak solutions to a novel two-phase model for various electrokinetic phenomena, including in particular dynamic electrowetting with electrolytes. The model is thermodynamically consistent and allows for contact angle hysteresis. It combines Navier-Stokes and Cahn-Hilliard type-phase-field equations with Nernst-Planck equations for ion density-evolution and with an elliptic transmission problem for the electrostatic potential. An interesting feature is the strong coupling between the equations for electrostatic potential and for phase-field and ion densities – the field intensity enters the chemical potential quadratically. As ion densities are a priori only bounded in $L \log L$-Orlicz-classes, a new iteration method is proposed to establish higher regularity and integrability of these quantities.
This is joint work with M. Fontelos (Madrid) and S. Jörres (Erlangen).

10.00, Tuesday, November 30, 2010.

Danielle Hilhorst

Université Paris - Sud, Orsay, France

title **A nonlinear cross-diffusion system for contact inhibition of cell growth**.

abstract We consider a parabolic-hyperbolic system of nonlinear partial differential equations which describes a simplified model for contact inhibition of growth of two cell populations. In one spatial dimension it is known that global solutions exist and that they satisfy the segregation property which reflects the inhibition mechanism: if initially the two populations are segregated - in mathematical terms this translates in (essentially) disjoint spatial supports of their densities - this property remains valid for all later times. The space-time curves which separate the two populations are free boundaries. We use recent results on transport quations and Lagrangian flows to obtain similar results in the case of several spatial variables.

Joint work with M. Bertsch, H. Izuhara and M. Mimura.

14.30, Monday, November 29, 2010.

Peter Knabner

Universität Erlangen - Nürnberg, Germany

title **Hierarchical and adaptive concept for the identification of material laws by boundary observations**.

abstract Identification of material laws (either as formfree nonlinearities or as parameters in a fixed (global) parametrization) from boundary state observations by means of an output least squares approach (OLS) requires specific means to detect the possible level of accuracy of reconstruction depending on the experimental design and the error level of the observation. We propose two means and their possible combination. Firstly, we use an unbiased parametrization of the unknown nonlinear functions, which does not employ any a-priori shape information. Piecewise polynomial functions provide an ansatz space where the unknown parameters only locally influence the representation of the nonlinearity. The minimization of the OLS-functional is highly sensitive to the initial value and slow convergence for a high number of degrees of freedom is to be expected. Therefore the identification is embedded into a multi-level algorithm which adapts the degree of parametrization to severeness of ill-posedness, as a low degree of parametrization acts as regularization. A stopping rule is based on the maximal error amplification according to the sensitivity matrix. Secondly, the problem of strongly varying sensitivities (even for given global parametrizations) has to be addressed. We propose an adaptive weighting in the OLS-functional based on the pseudoinverse of the sensitivity matrix at the achieved parameter set. The further nonlinearity, introduced into the OLS-functional is resolved in an retarding manner leading to a sequence of "usual" OLS-problems. The concepts are illustrated by outflow and breakthrough experiments with soil columns to identify either retention curve and unsaturated conductivity on reaction (Monod-) parameters for reactive transport.

10.45, Tuesday, November 30, 2010.

Angel Manuel Ramos

Universidad Complutense de Madrid, Spain

title **Modelling and simulation of high pressure processes in food engineering and corresponding inverse problems**.

abstract Nowadays, in industrialized countries, food products that are frequently consumed are processed in order to prolong their shelf life, to avoid as much as possible their decomposition, and to maintain or even improve their natural qualities such as flavor and color. Decomposition of food is mainly due to microorganisms and enzymes, since they are involved in the physical and chemical processes of transformation of food substances. At present, consumers look for minimally processed, additive-free food products that maintain their organoleptic properties. This has promoted the development of new technologies for food processing. One of these new emerging technologies is high hydrostatic pressure, as it has turned out to be very effective in prolonging the shelf life of foods without losing its properties.
This work deals with the modelling and simulation of the effect of the combination of Thermal and High Pressure Processes, focusing on the inactivation that occurs during the process of certain enzymes and microorganisms that are harmful to food. We propose various mathematical models that study the behavior of these enzymes and microorganisms during and after the process, and study some related inverse problems.

12.30, Monday, November 29, 2010.

Flavia Smarrazzo

"Sapienza" Università di Roma, Italy

title **Large-time behaviour of weak entropy measure-valued solutions to a class of equations with variable parabolicity direction**.

abstract In this talk I consider the Neumann initial-boundary value problem for the equation $u_t = \Delta\phi(u)$, where the nonlinear term ϕ is a cubic-like function. First I introduce the viscous pseudoparabolic regularization - described by the Sobolev equation $u_t = \Delta\phi(u) + \epsilon\Delta u_t$ - which leads to a family of well-posed approximating problems. Then I describe the vanishing viscosity limit in such regularized problems, defining in this way the class of weak entropy measure-valued solutions to the original unperturbed problem. Finally, in the one-dimensional case, the characterization of the large-time behaviour of such solutions is also given.

12.45, Tuesday, November 30, 2010.

Johannes Zimmer

University of Bath, United Kingdom

title **Ill-posed problem in nonlinear elasticity: Macro and micro**.

abstract The equations of elasticity in one space dimension, $u_{tt} = \sigma(u_x)_x$, become ill-posed if the potential energy density is nonconvex, or equivalently if σ is non-monotone. This complication necessarily arises in the theory of so-called martensitic phase transitions, which are diffusionless solid-solid transformations with where several stable phases can coexist.

Different regularisations of this ill-posed problem have been proposed; we will here focus on so-called kinetic relations, which relate the velocity of a moving interface to a driving force. Phenomenological kinetic relations have been proposed, but a natural question is whether they can in simple situations be derived from first principles, namely atomistic considerations.

To investigate this question, we study the simplest one-dimensional chain model of martensitic materials, where neighbouring atoms are coupled by a spring with bi-quadratic potential. We prove the existence of travelling waves and discuss a microscopic ill-posednedness that raises, namely the non-uniqueness of microscopic solutions. This non-uniqueness will be discussed in light of the macroscopic kinetic relation.

15.30, Tuesday, November 30, 2010.