The role of the model validation function to manage and mitigate model risk

Alberto Elices
Head of Equity Model Validation
Risk Methodology
Santander

Outline

- Introduction
- Taxonomy of model sources of uncertainty.
- Managing and mitigating model risk.
- Worked case examples.
- Conclusions.
Introduction

- After 2008 crisis, a big concern about pricing models has been raised.
- Risk management and model validation have drawn considerably more attention.
- Validation is not only about testing implementation but model adequacy and limitations.
- Simpler products & more complex model assumptions.
- Improving models is about improving what it’s around:
  - Improvement of quality of input market data.
  - Quality of output data and proper integration with risk management systems.
Taxonomy of model sources of uncertainty

- **Sources of uncertainty:**
  - Bad implementation.
  - Wrong use of model.
  - Uncertain model parameters.
  - Inconsistent market data.
  - Evolution of market consensus.
  - Missing key sources of risk.

- Some filtered after validation process.
- Others there, even for unimprovable models in practice.
- Use of limited models under controllable risk.
Taxonomy of model sources of uncertainty: Fixed Income

- Model risk:
  - Negative rates: review of lognormality of interest rates.
  - Autocorrelation among indexes: Libor, deferred Libor, Swap, CMS.
  - Lack of stochastic basis in multi-curve modelling.
  - Calibration risk: construction, interpolation and extrapolation of implied volatility surface.

- Other sources of risk:
  - Estimation of market data (e.g. correlation rates-credit and quanto).
  - Management of high change of vega with rate levels for notional increasing or accreting bermudan swaptions.
Taxonomy of model sources of uncertainty: Equity

- **Model risk:**
  - Forward skew for autocallable, barrier and cliquet options: (not clear benchmark, not clear model parameters).
  - Impact of stochastic rates in autocallable and barrier products.
  - Impact of multi-curve modelling.
  - Marking correlation level and considering correlation skew.

- **Other sources of risk:**
  - Management cost of liquidity: maximum delta softening for digital, callable, barrier and gamma softening for cliquet options.
  - Management of cross-gamma.
  - Management of skew movements through vega maps.
  - Estimation of dividends and quanto and composite correlations.
Taxonomy of model uncertainty: Foreign exchange & Inflation

- **Foreign exchange:**
  - Estimation of forward skew for barrier options:
    (benchmark -> Stochastic Local Volatility, uncertain model params)
  - Impact of stochastic rates.
  - Correlation implied from volatility surface building of illiquid pairs.

- **Inflation:**
  - Lack of considering volatility smile.
  - Correlation structure among year-on-year CPI returns.
  - Estimation of correlation between inflation and interest rates.
Managing and mitigating model risk

- **Periodic review of pricing models:**
  - Inventory, classification of products, models and engines, decommissioning old ones.

- **Qualitative rating of models:**
  - Rate products and models to make model users and senior management aware of product and model risk (e.g. traffic light).

- **Valuation control policy:**
  - Quality of market data: is market data used for calibration reasonable (e.g. arbitrage free)?
  - Are non-calibrated model parameters providing prices according to market consensus (e.g. Markit consensus)?
  - How well are collateral disputes explained?
Managing and mitigating model risk

- **Mitigation policies:**
  - Set limits to some sensitivities: dividends, correlation, cross-gamma, vanna.
  - Reduce hedging error using maximum delta and gamma softening.

- **Fair value adjustment (FVA).**
Fair Value Adjustment: Managing and mitigating model risk

- **FVA: How to reconcile FO and Risk interests?:**
  - FVA should cover expected hedging loss \((V_{fair}^{mkt} - V_{Hedge}^{model})\) + uncertainty.
  - FVA allows using limited models with controllable risk.
  - FVA fosters model improvement: better models imply lower FVA.
  - How should FVA be? dynamic, stable, transparent, easy to compute, lower with lower uncertainty and better quality of model.
External Fair value adjustment

- **Comparison of a product priced with different models:**
  - Models for FVA calculation should be integrated in production:
    a. LV vs SLV for barriers, autocallables.
    b. HW vs normal SABR for zero strike floors.
    c. HW vs HJM for callable range accrual.

- **Comparison of same model varying internal params:**
  - Parameters varied for a percentile range of a historical distribution, various calibration sets, range to fit market consensus:
    a. SLV varying stochastic volatility parameters for cliquet options.
    b. HJM varying calibration sets for callable range accrual.
    c. HW varying mean reversion to fit autocorrelation.

- **A factor times sensitivity to unobserved mkt params:**
  - Sensitivity to 1% change of dividends, correlation, cross-gamma.
Internal Fair value adjustment

- Deals booked with conservative internal parameters:
  - Positions marked with e.g. correlation, mean reversion, stochastic volatility parameters, providing conservative pricing (e.g. from historical distributions with a conservative percentile).

- Deals booked with conservative softening.
Internal Fair value adjustment

- External versus internal FVA:
  - External FVA allows marking books to market, avoiding collateral disputes and better Risk control.
  - Internal FVA may ease position management: e.g. excess of softening makes sensitivities smoother and easier to manage.
  - **Not a good practice to embed FVA for wrong concepts:** e.g. excess of softening to account for forward skew instead of liquidity. Uncertainty of hedging error increases.
  - FVA tradeoff: a part of FVA internal and the rest external?
Outline

- Introduction
- Taxonomy of model sources of uncertainty.
- Managing and mitigating model risk.
- Worked case studies.
- Conclusions.
Case I: FVA for autocallables (*)

Standard model: LV?

Client receives call Coupons if $S(t_i) >$ Cancellation Trigger

Client pays floating leg payments (SWAP) until call event

Client pays Put DI at maturity if not called

Client receives contingent coupons when $S(t_i) >$ Trigger

Standard structure

(*) Study performed by D. Enes, Global Derivatives 2013, Santander
Case I: FVA for autocallables: qualitative impact

- **Effect of stochastic interest rates (LVHW vs LV):**

\[
\sigma_{LV}^2 = \int_0^T \sigma_s^2 + 2\left(\frac{1 - e^{-a(T-s)}}{a}\right)\sigma_s \rho \cdot \tau^2 \left(\frac{1 - e^{-a(T-s)}}{a}\right)^2 ds
\]

- Total variance taken from european prices
- Underlying’s variance
- Mixed contribution from both stoch. factors, weighted by the correlation
- Rates variance

- **Effect of stochastic volatility (SLV vs LV):**

1y SLV impl vol, 1y fwd

1y LV impl vol, 1y fwd
Case I: FVA for autocallables: worked example

- Basic Structure for comparison
  - Maturity: 5 years
  - Call condition observed: yearly
  - Call condition: $\geq 100\% S_{\text{Initial}}$
  - If not called: $-S_{\text{Final}} / S_{\text{Initial}}$, if $S_{\text{Final}} \leq 50\% S_{\text{Initial}}$
  - Call Coupons: $X\%$, solved to have 100% issue price
  - Call Coupons: $n \times X\%$, $n = 1, 2, \ldots$ on each date

- Starting LVHW parameters: $\rho = 0.3$ and $\tau = 0.5\%$.

- SLV parameters:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>MeanRev</th>
<th>VolVol</th>
<th>Corr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>4</td>
<td>1.2</td>
<td>-0.6</td>
</tr>
<tr>
<td>Adding skew</td>
<td>4</td>
<td>1.2</td>
<td>-0.9</td>
</tr>
<tr>
<td>Homogenizing skew</td>
<td>4</td>
<td>1.8</td>
<td>-0.9</td>
</tr>
</tbody>
</table>
Case I: FVA for autocallables: impact of stochastic rates

Contingent Coupons & KI Level: the reduction of volatility decreases barrier touching probability, increasing the price

<table>
<thead>
<tr>
<th>Contingent Coupons Trigger</th>
<th>80%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVHW - LV (bps)</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>increasing rates vol. (τ = 1.2%)</td>
<td>31</td>
<td>40</td>
</tr>
<tr>
<td>increasing correl. (ρ = 0.5)</td>
<td>53</td>
<td>65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>KI Level</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVHW - LV (bps)</td>
<td>5</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>increasing rates vol. (τ = 1.2%)</td>
<td>9</td>
<td>17</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>increasing correl. (ρ = 0.5)</td>
<td>11</td>
<td>21</td>
<td>35</td>
<td>44</td>
</tr>
</tbody>
</table>

Capital Guaranteed (without Put DI): payoff goes in one direction, positive correlation increases even more the price.

<table>
<thead>
<tr>
<th>Capital Guaranteed</th>
<th>-0.5</th>
<th>-0.3</th>
<th>0</th>
<th>0.3</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2y</td>
<td>-14</td>
<td>-9</td>
<td>1</td>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>3y</td>
<td>-35</td>
<td>-21</td>
<td>1</td>
<td>23</td>
<td>38</td>
</tr>
<tr>
<td>5y</td>
<td>-78</td>
<td>-46</td>
<td>4</td>
<td>52</td>
<td>85</td>
</tr>
</tbody>
</table>

LVHW – LV price (in bps), changing maturity and correlation
Case I: FVA for autocallables: impact of forward skew

Skew effect

Call UO (SLV) < Call UO (LV)  
(SLV cancels less often than LV) but  
Put DI (SLV) << Put DI (LV)  
⇒ Autocall (SLV) > Autocall (LV)

Capital Guaranteed (no short put): we only have the impact of Call UO

<table>
<thead>
<tr>
<th>Capital Guaranteed</th>
<th>3y</th>
<th>5y</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLV mild skew - LV (bps)</td>
<td>-15</td>
<td>-21</td>
</tr>
<tr>
<td>adding skew</td>
<td>-17</td>
<td>-23</td>
</tr>
<tr>
<td>homogenizing the skew</td>
<td>-25</td>
<td>-37</td>
</tr>
</tbody>
</table>
Case I: FVA for autocallables: impact of forward skew

Moving Triggers, Contingent Coupons and KI Level: lowering the barrier capitalizes more on the skew effect

<table>
<thead>
<tr>
<th>Call Triggers</th>
<th>all at 100%</th>
<th>decreasing 10%/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLV mild skew - LV (bps)</td>
<td>62</td>
<td>102</td>
</tr>
<tr>
<td>adding skew</td>
<td>97</td>
<td>144</td>
</tr>
<tr>
<td>homogenizing the skew</td>
<td>133</td>
<td>208</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Contingent Coupons Trigger</th>
<th>80%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLV mild skew - LV (bps)</td>
<td>93</td>
<td>103</td>
</tr>
<tr>
<td>adding skew</td>
<td>143</td>
<td>151</td>
</tr>
<tr>
<td>homogenizing the skew</td>
<td>196</td>
<td>211</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>KI Level</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLV mild skew - LV (bps)</td>
<td>27</td>
<td>39</td>
<td>53</td>
<td>62</td>
</tr>
<tr>
<td>adding skew</td>
<td>52</td>
<td>68</td>
<td>82</td>
<td>97</td>
</tr>
<tr>
<td>homogenizing the skew</td>
<td>72</td>
<td>94</td>
<td>116</td>
<td>133</td>
</tr>
</tbody>
</table>

- Impact of forward skew seems more relevant than impact of stochastic rates.
Case II: Impact of stochastic basis on Bermudan swaptions (*)

- A Hull-White model is assumed for the discount and the spread curves:

\[
\begin{align*}
&dr_{d,t} = (\theta_{d,t} - a_{d,t}r_{d,t})dt + \sigma_{d,t}dW_{d,t}^P \\
&dr_{s,t} = (\theta_{s,t} - a_{s,t}r_{s,t})dt + \sigma_{s,t}dW_{s,t}^P \\
&\quad r_{e,t} := r_{d,t} + r_{s,t}
\end{align*}
\]

\[
f_{s,t} := \frac{\sigma_{s,t}}{b_{sd,t}} \\
b_{s,t} := e^{-\int_0^t a_{s,v}dv} \\
< dW_{d,t}^P, dW_{s,t}^P > = \rho dt
\]

- Externally input: \( a_{d,t} = 0.0001 \), \( \rho \) and \( f_{s,t} \)

- Calibration:
  - Replication of discount curves.
  - Anti-diagonal swaptions (e.g. for 10y: 1into9, 2into8, … , 9into1).
  - Strike of swaptions corresponding to Bermudan strike.

(*) Study performed by G. Montesinos, Santander
Case II: Impact of stochastic basis on Bermudan swaptions

- Maximum impact varying correlation $\rho$ and volatility $f_s$, for different strikes and mean reversion $\alpha_s$:

<table>
<thead>
<tr>
<th>$K \backslash \alpha_s$</th>
<th>0.0001</th>
<th>0.01</th>
<th>0.02</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5%</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>17</td>
<td>28</td>
<td>33</td>
</tr>
<tr>
<td>1.8%</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>16</td>
<td>27</td>
<td>31</td>
</tr>
<tr>
<td>2.0%</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>16</td>
<td>25</td>
<td>29</td>
</tr>
<tr>
<td>2.5%</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>14</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>3.0%</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>11</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>3.5%</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>10</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>4.0%</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

- Difference appears when mean reversion of basis increases.
Case III: Negative rates: Interest rate zero strike floor (*)

- 3y Vanilla floor on GBP LIBOR1M at zero strike with monthly caplets:
  - Product neither easy to observe nor to price test.
  - GBP LIBOR1M is very illiquid. Need to rely on GBP LIBOR3M.
  - With traditional lognormal hypothesis, this product has zero price.
  - Difficult to mark to market, therefore necessary to mark to model.

- Still not a clear consensus for negative rates:
  - **Hull-White model**: calibrated only to the lowest available strike strike and all caplet maturities.
  - **Normal SABR**: arbitrage free extrapolation of low floor strikes consistent with cap/floor smile for all maturities.
  - **Shifted log-normal**: provides lognormal behaviour only for a given negative range. Consensus is trending to this model.

(*) Study performed by M. Pardo and J. C. Esparragoza, Santander
Case III: Negative rates: Interest rate zero strike floor

- Calibration to caplets on GBPLIBOR3M and apply deterministic basis spread 1M-3M?
- Hull-White versus normal SABR:

Cumulative density function

Hull-White conservative usu. for caplets under 3Y.

Normal SABR, $\beta = 0$:

$$dF_t = \sigma_t \cdot F_t^\beta \cdot dW_t$$

$$d\sigma_t = \alpha \cdot \sigma_t \cdot dZ_t$$
Case III: Negative rates: Interest rate zero strike floor

- Normal vs lognormal vol (e.g. ICAP quotes both):
  \[ \sigma^N \approx \sigma^{LN} \cdot F_{ATM} \quad \text{vega}^{LN} \approx \text{vega}^N \cdot F_{ATM} \]

- Comparison 3y vanilla floor on GBPLIBOR1M at zero strike with monthly caplets:

<table>
<thead>
<tr>
<th></th>
<th>SABR</th>
<th>HW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium (bp)</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>Delta (bp)</td>
<td>-0.234</td>
<td>-0.159</td>
</tr>
<tr>
<td>VegaLN (bp)</td>
<td>0.248</td>
<td>0.631</td>
</tr>
<tr>
<td>VegaN (bp)</td>
<td>40.655</td>
<td>103.442</td>
</tr>
</tbody>
</table>

- Ways to mitigate model risk:
  - Price testing with floors struck up to 20 basis points.
  - Monitoring with respect to alternative models.
Case IV: Comparison of SLV and FSP models for cliquet valuation

- **Forward Skew Propagation model (FSP):** Forward skew is deterministic and usually set to spot skew.

\[
\Sigma(k, t_1, t_2 | S_{t_1}) \equiv \Sigma(k, t_1, t_2) = \sigma_{fwd, ATM} F(t_1, t_2) + \kappa [\sigma(S_0 \cdot k, t_2) - \sigma(S_0, t_2)] \cdot \left(\frac{t_2}{t_2 - t_1}\right)^\alpha
\]
Comparison of SLV versus FSP for skew cliquet products

- Stochastic Local Volatility model (SLV): forward skew is controlled by stochastic volatility parameters.

\[
\begin{align*}
\frac{dS_t}{S_t} &= \mu dt + \sigma_{SLV}(t, S_t) e^{x_t} dW_t \\
\frac{dx_t}{S_t} &= (-\kappa x - \frac{1}{2} \omega^2) dt + \omega dV_t \\
d\langle W_t, V_t \rangle &= \rho dt
\end{align*}
\]

1m implied volatility, 1y forward, varying mean reversion, $\kappa$. 

![Graphs showing the comparison of SLV versus FSP for skew cliquet products.](image)
Comparison of SLV versus FSP for skew cliquet products

- Reasonable agreement of premium and sensitivities:
  - 3y cliquet options, zero local and global floor, varying local cap and resetting period: (1y, 7%), (6m, 3.5%), (3m, 1.75%) and (1m, 0.58%).
  - Same SLV and FSP parameters for all cases except for 1m resetting with lower FSP alpha, as SLV is unable to reach same FwdSkew.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Premium bp</th>
<th>Delta bp per unit spot change</th>
<th>Gamma bp per 1% spot change</th>
<th>Vega bp per 1% vol change</th>
</tr>
</thead>
<tbody>
<tr>
<td>3yCliq1y</td>
<td>FSP</td>
<td>200.5</td>
<td>0.0280</td>
<td>-0.0002</td>
<td>2.32</td>
</tr>
<tr>
<td>C: 7%</td>
<td>SLV</td>
<td>203.5</td>
<td>0.0294</td>
<td>0.0002</td>
<td>1.94</td>
</tr>
<tr>
<td>3yCliq6m</td>
<td>FSP</td>
<td>161.7</td>
<td>0.0470</td>
<td>-0.0011</td>
<td>2.81</td>
</tr>
<tr>
<td>C: 3.5%</td>
<td>SLV</td>
<td>164.7</td>
<td>0.0509</td>
<td>0.0006</td>
<td>2.33</td>
</tr>
<tr>
<td>3yCliq3m</td>
<td>FSP</td>
<td>136.8</td>
<td>0.0245</td>
<td>-0.0008</td>
<td>1.86</td>
</tr>
<tr>
<td>C: 1.75%</td>
<td>SLV</td>
<td>134.6</td>
<td>0.0292</td>
<td>-0.0015</td>
<td>1.58</td>
</tr>
<tr>
<td>3yCliq1m</td>
<td>FSP</td>
<td>99.8</td>
<td>0.0066</td>
<td>-0.0008</td>
<td>1.31</td>
</tr>
<tr>
<td>C: 0.85%</td>
<td>SLV</td>
<td>92.0</td>
<td>0.0108</td>
<td>-0.0001</td>
<td>1.24</td>
</tr>
</tbody>
</table>
Comparison of SLV versus FSP for skew cliquet products

- Reasonable agreement varying spot level and applying parallel volatility movements:

  3y Cliquet, 1y resetting, LF:0%, LC: 7%, GF: 0%
Comparison of SLV versus FSP for skew cliquet products

- There are differences:
  - FSP: moving spot skew moves forward skew. SLV: not as much.
  - Sensitivity to skew: lower SLV sensitivity for shorter resetting.
  - Sensitivity to convexity: generally low and lower in FSP.
  - Combined movements: almost reduced to skew movements.
Conclusions

- Current trend of pricing models evolves towards simpler products and more complex model assumptions.

- Model improvements are:
  - Less about developing new models.
  - More about improving existing ones: performance, inputs and outputs and good integration in risk management systems.

- Model validation function comes into play to:
  - Guarantee permanent improvement of pricing models.
  - Correct management and mitigation of model risk.
  - Making senior management aware of model risk.