Perturbed copula: Introducing the skew effect in the co-dependence

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Outline

- Introduction.
- Formulation of perturbed copula.
- Calibration.
- Interpretation.
- Case study: FX quanto options to third currency.
  - Comparison between gaussian and perturbed copulas.
  - Comparison with a market standard: local volatility model.
- Conclusions.
Introduction

- Copula models are widely used to obtain joint asset behaviour when market information is only available for single assets.

- It is necessary to make assumptions about joint relationships when liquid multi-asset products are not available or their joint relationships are very complex.

- Most widely used copula model: gaussian copula.
  - Advantages: It is analytical, easy to interpret and condenses co-dependence in a single parameter: the correlation.
  - Drawbacks: *no fat tail behaviour*, own asset information is not incorporated in the co-dependence.
Contributions of perturbed copula:
- Incorporate the skew effect in the co-dependence of two lognormal-inspired random variables.
- Hypothesis: both random variables have a common fast mean-reverting stochastic volatility factor.
- The perturbed copula is analytic: both joint and marginal distributions are approximated using asymptotic expansions.
- The copula incorporates two parameters for each asset (level and slope of skew) as well as correlation.

Examples of traditional applications of copulas:
- Credit models (joint distribution of many factors).
- Hybrid or exotic models (joint distributions of a few factors).
Introduction

- **Copula theory:**
  - Marginal distributions $f_i^{\text{marg}}(x_i)$ of values $x_i$ are known.
  - Original variables $x_i$ are normalized into $z_i$ confined in [0,1] through the cumulative density function.

\[
z_i = \int_{-\infty}^{x_i} f_i^{\text{marg}}(s) ds
\]

- The joint density shows how co-dependence, $f_{\text{cop}}(z_1, z_2)$, can be isolated from marginal behaviour $f_i^{\text{marg}}(x_i)$

\[
f_i^{\text{joint}}(x_1, x_2) = f_{\text{cop}}(z_1, z_2) f_1^{\text{marg}}(x_1) f_2^{\text{marg}}(x_2)
\]

- Copula function has the whole co-dependence information:

\[
f_{\text{cop}}(z_1, z_2) = \frac{f_{\text{cop}}^{\text{joint}}(\xi_1, \xi_2)}{f_{\text{cop} 1}^{\text{marg}}(\xi_1) f_{\text{cop} 2}^{\text{marg}}(\xi_2)} \quad z_i = \int_{-\infty}^{\xi_i} f_{\text{cop} i}^{\text{marg}}(s) ds
\]
Formulation of perturbed copula

- Starting point: lognormal-inspired underlyings with vols related by a fast mean reverting ($\varepsilon \to 0$) O-U process.

\[
\begin{align*}
   dX_t^{(1)} &= \left(\alpha_t^{(1)} - \frac{1}{2} f_1^2 (Y_t)\right) dt + f_1 (Y_t) dW_t^{(1)} \\
   dX_t^{(2)} &= \left(\alpha_t^{(2)} - \frac{1}{2} f_2^2 (Y_t)\right) dt + f_2 (Y_t) dW_t^{(2)} \\
   dY_t &= \frac{1}{\varepsilon} (m - Y_t) dt + \frac{\nu \sqrt{2}}{\sqrt{\varepsilon}} dW_t^{(Y)} \\
   X_t^{(i)} &= \ln\left(S_t^{(i)}\right)
\end{align*}
\]

- Co-dependence defined by $f_i (Y_t)$ and correlations: $\rho \rho_{1Y} \rho_{2Y}$

\[
\begin{align*}
   d(W_t^{(1)}, W_t^{(2)}) &= \rho dt \\
   d(W_t^{(1)}, W_t^{(Y)}) &= \rho_{1Y} dt \\
   d(W_t^{(2)}, W_t^{(Y)}) &= \rho_{2Y} dt
\end{align*}
\]
Formulation of perturbed copula

- Additional hypothesis: common, normalized ($\langle g^2 \rangle = 1$) volatility dynamics with different levels ($\sigma_1$ & $\sigma_2$)

$$f_1(y) = \sigma_1 g(y) \quad f_2(y) = \sigma_2 g(y) \quad \langle g \rangle = \int_{-\infty}^{\infty} g(y) \frac{1}{\nu \sqrt{2\pi}} \exp\left(-\frac{(y - m)^2}{2\nu^2}\right) dy$$

- Objective: know copula joint & marginal distributions:

$$u^\xi = P\left( X_T^{(1)} \in d\xi_1, X_T^{(2)} \in d\xi_2 | X_t = x, Y_t = y \right)$$

$$v_1^\xi = P\left( X_T^{(1)} \in d\xi_1 | X_t = x_1, Y_t = y \right)$$

$$v_2^\xi = P\left( X_T^{(2)} \in d\xi_2 | X_t = x_2, Y_t = y \right)$$

- Copula function:

$$f_{cop} (z_1, z_2) = \frac{u^\xi(t, x_1, x_2; T, \xi_1, \xi_2)}{v_1^\xi(t, x_1, T, \xi_1) v_2^\xi(t, x_2, T, \xi_2)}$$

$$z_i = P\left( X_T^{(i)} \leq \xi_i | X_t = x, Y_t = y \right)$$
Formulation of perturbed copula

- The joint and marginal transition density functions satisfy the F-P (Fokker-Plank) equation:

\[
\mathcal{L}^\varepsilon u^\varepsilon(t, x_1, x_2, y) = 0 \\
u^\varepsilon(T, x_1, x_2, y) = \delta(\xi_1; x_1)\delta(\xi_2; x_2)
\]

\[
\mathcal{L}^\varepsilon = \frac{1}{\varepsilon}\mathcal{L}_0 + \frac{1}{\sqrt{\varepsilon}}\mathcal{L}_1 + \mathcal{L}_2
\]

\[
\mathcal{L}_0 = (m - y) \frac{\partial}{\partial y} + \nu^2 \frac{\partial^2}{\partial y^2}
\]

\[
\mathcal{L}_1 = \nu \sqrt{2}\rho_{1Y} f_1(y) \frac{\partial^2}{\partial x_1 \partial y} + \nu \sqrt{2}\rho_{2Y} f_2(y) \frac{\partial^2}{\partial x_2 \partial y}
\]

\[
\mathcal{L}_2: \text{Fokker-Plank generator for two underlyings.}
\]
The solution is expanded in powers of \( \sqrt{\varepsilon} \):

\[
u^\varepsilon = u_0 + \sqrt{\varepsilon}u_1 + \varepsilon u_2 + \varepsilon^{3/2}u_3 + \cdots
\]

Replacing the expansion in the F-P equation, a system of PDEs is obtained.

\[
\frac{1}{\varepsilon}\mathcal{L}_0 u_0 + \frac{1}{\sqrt{\varepsilon}}(\mathcal{L}_0 u_1 + \mathcal{L}_1 u_0) + (\mathcal{L}_0 u_2 + \mathcal{L}_1 u_1 + \mathcal{L}_2 u_0) + \\
+ \sqrt{\varepsilon}(\mathcal{L}_0 u_3 + \mathcal{L}_1 u_2 + \mathcal{L}_2 u_1) + \cdots = 0
\]

\[
\begin{cases}
\mathcal{L}_0 u_0 = 0 \\
\mathcal{L}_0 u_1 + \mathcal{L}_1 u_0 = 0 \\
\mathcal{L}_0 u_2 + \mathcal{L}_1 u_1 + \mathcal{L}_2 u_0 = 0 \\
\mathcal{L}_0 u_3 + \mathcal{L}_1 u_2 + \mathcal{L}_2 u_1 = 0
\end{cases}
\]

This system of PDEs is solved up to first order: \( u_0, \sqrt{\varepsilon}u_1 \)
Formulation of perturbed copula

- Zero order solution: joint density of gaussian copula:

\[
u_0 = \frac{1}{2\pi \sigma_1 \sigma_2 (T-t) \sqrt{1-\rho^2}} \exp \left( -\frac{1}{2(1-\rho^2)} \left[ \frac{(\xi_1 - \tilde{x}_i)^2}{\sigma_1^2 (T-t)} - 2\rho \frac{(\xi_1 - \tilde{x}_1)(\xi_2 - \tilde{x}_2)}{\sigma_1 \sigma_2 (T-t)} + \frac{(\xi_2 - \tilde{x}_2)^2}{\sigma_2^2 (T-t)} \right] \right) \]

\[
\tilde{x}_i = x_i + \int_t^T \alpha_s^{(i)} ds - \frac{1}{2} \sigma_i^2 (T-t)
\]

- First order solution: obtained by derivatives of \( u_0 \) plus some coefficients calibrated to market: \( R_1, R_2 \)

\[
\sqrt{\varepsilon} u_1 = - (T - t) \left\{ R_1 \left( \frac{\partial^3 u_0}{\partial x_1^3} - \frac{\partial^2 u_0}{\partial x_1^2} \right) + R_2 \left( \frac{\partial^3 u_0}{\partial x_2^3} - \frac{\partial^2 u_0}{\partial x_2^2} \right) \\
+ R_{12} \frac{\partial^3 u_0}{\partial x_1 \partial x_2^2} + R_{21} \frac{\partial^3 u_0}{\partial x_2^2 \partial x_1} - (Q_{12} + Q_{21}) \frac{\partial^2 u_0}{\partial x_1 \partial x_2} \right\}
\]

\[
R_{ij} = \left( \frac{\sigma_j}{\sigma_i} \right)^2 R_i + 2 \left( \frac{\sigma_j}{\sigma_i} \right) R_j \rho \\
Q_{ij} = \left( \frac{\sigma_j}{\sigma_i} \right)^2 R_i
\]
Formulation of perturbed copula

- To get positive densities: solution in terms of tanh:

\[
u^{\varepsilon}(t, x_1, x_2; T, \xi_1, \xi_2) = \frac{1}{W} u_0 \left\{ 1 + \tanh \left( \frac{\sqrt{\varepsilon} u_1}{u_0} \right) \right\}
\]

- Similar solution for marginal densities:

\[
p_i(t, x_i; T, \xi_i) = \frac{1}{\sqrt{2\pi(T-t)}} \exp \left( -\frac{(\xi_i - \tilde{x}_i)^2}{2\sigma_i^2(T-t)} \right)
\]

\[
\tilde{x}_i = x_i + \int_t^T \alpha_s^{(i)} ds - \frac{1}{2} \sigma_i^2(T-t)
\]

\[
v_i^{\varepsilon}(t, x_1; T, \xi_i) = \frac{1}{W_i} p_i \left[ 1 + \tanh \left( -(T - t) \frac{R_i}{p_i} \left\{ \frac{\partial^3 p_i}{\partial x_i^3} - \frac{\partial^2 p_i}{\partial x_i^2} \right\} \right) \right]
\]
- **Interpretation of the 5 parameters of the copula:**
  - $\sigma_1$: controls the volatility level of $S_1$.
  - $\sigma_2$: controls the volatility level of $S_2$.
  - $R_1$: controls the skew of $S_1$.
  - $R_2$: controls the skew of $S_2$.
  - $\rho$: controls the correlation between $S_1$ and $S_2$. 
**Calibration**

- $R_i$ and $\sigma_i$ are calibrated to fit exactly 2 strikes of each underlying skew using Newton-Raphson.

$$c_i(K,T) = P(t, T) \int_{-\infty}^{\infty} (\exp(\beta_i + \xi_i) - K)^+ v_i^\epsilon(t, 0, T, \xi_i) \, d\xi_i$$

- An initial guess for the parameters is estimated using a similar asymptotic procedure. $\rho$ is input externally.

$$\sigma_{i0}^{\text{impl}}(K) = a \left\{ \ln \left( \frac{K}{F_{iT}} \right) \right\} + b$$

$$\beta_{i0} = \ln S_0^{(i)} + \int_t^T (r_s^{(i)} - q_s^{(i)}) \, ds - \frac{1}{2} \sigma_i^2 (T-t)^2$$

$$\sigma_{i0} = b - \frac{ab^2}{2} \quad R_{i0} = -a \sigma_i^3$$

*a & b are obtained through linear regression of implied vol onto log-moneyness-to-maturity ratio.*
Calibration: left and right skew generated with a Heston model

- Left: fitting of implied volatility: original vs perturbed copula.
- Mid: marginal copula density for left and right skew (LS, RS)
- Right: empirical density for left and right skew.
Interpretation

- Valuation of FX call on XAU/USD quantoed to EUR. Both $S$ (XAU/USD) and $X$ (EUR/USD) are quoted in USD.

\[ p = E \left[ (S_T - K)^+ X_T D_{USD}F_T \right] \]

- Double integral valuation with USD numeraire:

\[ p = D_{USD}F_T \int_{\mathbb{R}^2} \left( e^{x_1} - K \right)^+ e^{x_2} f_{\text{joint}}(x_1, x_2) dx_1 dx_2 \]

\[ f_{\text{joint}}(x_1, x_2) = f_{\text{cop}}(z_1, z_2) f_{1_{\text{marg}}}(x_1) f_{2_{\text{marg}}}(x_2) \]

\[ f_{i_{\text{marg}}}(x_i) = \frac{1}{D_{USD}F_T} \frac{\partial^2 P_i(e^{x_i}, T, \sigma_i^{\text{impl}}(e^{x_i}, T))}{\partial K^2} \]

- 5 Heston skew scenarios LR, RL, RR & LL are created:
  - LL is shown; the rest can be obtained through symmetries.
Interpretation: left-left skew (LL): copula related functions

- Left: copula joint density function: \( f_{cop}^{\text{joint}} = u^\epsilon \)
- Middle: copula function \( f_{cop} = u^\epsilon / (u_1^\epsilon \cdot u_2^\epsilon) \) greater than one.
- Right: ratio > 1 of perturbed over gaussian copula functions.

\[ \rho = +0.6 \]
\[ \rho = -0.6 \]
Interpretation: left-left skew (LL): real joint densities

- **Left:** joint density \( f_{\text{joint}} = f_{\text{cop}} \cdot f_{\text{marg}} \cdot f_{\text{marg}} \) using *gaussian* copula.
- **Mid:** joint density \( f_{\text{joint}} = f_{\text{cop}} \cdot f_{\text{marg}} \cdot f_{\text{marg}} \) using *perturbed* copula.
- **Right:** ratio > 1 of perturbed over gaussian joint density.

\[ \rho = +0.6 \]

\[ \rho = -0.6 \]
Interpretation: premium corrections

\[ p = \mathbb{E} \left[ (S_T - K)^+ X_T DF_T^{USD} \right] \]

<table>
<thead>
<tr>
<th></th>
<th>Sce</th>
<th>Gcop</th>
<th>Pcop</th>
<th>P-G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-LR ( \rho = +0.6 )</td>
<td>0.2640</td>
<td>0.2671</td>
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<td>17-RR ( \rho = +0.3 )</td>
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<td>-0.0026</td>
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<td>0.2489</td>
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<td>21-LL ( \rho = +0.6 )</td>
<td>0.2642</td>
<td>0.2662</td>
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<td>22-LL ( \rho = +0.3 )</td>
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<td>0.2509</td>
<td>0.0030</td>
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</tbody>
</table>

Joint copula functions
Case Study: FX quanto options to a third currency

- Valuation of FX call on XAU/USD quantoed to EUR. Both S (XAU/USD) and X (EUR/USD) are quoted in USD.
  - 5 correlation scenarios considered (0.6, 0.3, 0, -0.3, -0.6).
  - XAU/USD is highly left-skewed (OTM puts are favored).
  - EUR/USD is very mildly right-skewed (almost a smile).
  - \( \text{Spot}_{\text{XAU/USD}} = 937.79 \quad \text{Spot}_{\text{EUR/USD}} = 1.4029 \).
  - Strikes used: (656.46, 797.13, 937.79, 1078.47, 1125.36).
  - Maturities: 1y, 2y.
Case Study: Calibration to left skew – smile (LS) scenario

- Left: fitting of implied volatility: original vs perturbed copula.
- Mid: marginal copula density for left skew & smile (LS, SML)
- Right: empirical density for left skew and smile.
Case Study: left skew-smile (LS): copula related functions

- Left: copula joint density function: \( f_{\text{cop}}^{\text{joint}} = u^\varepsilon \)
- Middle: copula function \( f_{\text{cop}} = u^\varepsilon / (u_1^\varepsilon \cdot u_2^\varepsilon) \) greater than one.
- Right: ratio > 1 of perturbed over gaussian copula.

\[ \rho = \pm 0.6 \]
Case Study: left skew-smile (LS): real joint densities

- Left: joint density \( f_{\text{joint}} = f_{\text{cop}} f_{1 \text{marg}} f_{2 \text{marg}} \) using gaussian copula.
- Mid: joint density \( f_{\text{joint}} = f_{\text{cop}} f_{1 \text{marg}} f_{2 \text{marg}} \) using perturbed copula.
- Right: ratio > 1 of perturbed over gaussian joint density.

\[ \rho = +0.6 \quad \rho = -0.6 \]
Case Study: comparison of gaussian versus perturbed copula

- **Premium difference in bp of perturbed minus gaussian copula varying moneyness, maturity & correlation:**
  - All corrections are *positive* and can go beyond 1% of notional amount.

<table>
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<tr>
<th>Sce — corr</th>
<th>+0.6</th>
<th>+0.3</th>
<th>+0.0</th>
<th>-0.3</th>
<th>-0.6</th>
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<tr>
<td>0.7 1y</td>
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<td>0.85 1y</td>
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<td>0.7 2y</td>
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Case Study: comparison of gaussian copula versus local volatility model

- Premium differences in bp between gaussian copula and a local volatility model with constant correlation:
  - Gaussian copula is almost equivalent to local volatility with constant correlation.

<table>
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<th>Scen</th>
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<td>-8.8819</td>
<td>-8.6984</td>
<td>-6.6643</td>
</tr>
</tbody>
</table>
Conclusions

- The perturbed copula has been successfully applied to introduce the skew information in the co-dependence:
  - For widely used lognormal-inspired underlyings.
  - In terms of simple very intuitive parameters: slope of skew and volatility level of each underlying plus correlation.
- An exact fit calibration of the copula parameters using N-R with a good initial point estimation is proposed.
- A qualitative interpretation of the perturbed copula action compared to the gaussian copula is provided.
  - How skew direction and magnitude affects co-dependence.
  - How the end joint distribution is affected qualitatively.
  - How the final premium of a given payoff might be affected.
Conclusions

- A real market case study for FX quanto options to a third currency is presented:
  - The impact of skew in the co-dependence is not negligible. It can go beyond 1% of the notional amount.
  - Valuation with gaussian copula is almost equivalent to valuation with local volatility model with constant correlation.

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