Hyperbolicity in the Gromov sense of the strong product of two graphs.

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Abstract

If $X$ is a geodesic metric space and $x_1, x_2, x_3 \in X$, a geodesic triangle $T = \{x_1, x_2, x_3\}$ is the union of the three geodesics $[x_1x_2]$, $[x_2x_3]$ and $[x_3x_1]$ in $X$. The space $X$ is $\delta$-hyperbolic (in the Gromov sense) if any side of $T$ is contained in a $\delta$-neighborhood of the union of the other two sides, for every geodesic triangle $T$ in $X$. In this paper we characterize the strong products of two graphs $G_1 \boxtimes G_2$ which are hyperbolic, in terms of $G_1$ and $G_2$: the strong product graph $G_1 \boxtimes G_2$ is hyperbolic if and only if one of the factors is hyperbolic and the other one is bounded. We also prove some sharp relations between $\delta(G_1 \boxtimes G_2)$, $\delta(G_1)$, $\delta(G_2)$ and the diameters of $G_1$ and $G_2$. 