FOREWORD

Twentieth century mathematicians have successfully mastered a method or way of looking that combines local and global tools, and which has lead, for example, to the resolution of long standing open problems such as Fermat’s Last Theorem and the Conjecture of Taniyama, Shimura and Weil. In parts 1 and 2 we will give a brief description of this way of looking, which we will then use, in part 3 as well as in the appendix, to analyse three concrete paintings of Pablo Picasso.

When Michele Emmer invited me to participate in the second volume of “The Visual Mind”, I was already deeply immersed in an on-going process that has been moving quite different people -most of us mathematicians, painters, musicians and architects-, for a long time. This process has lead us to analyze, in conversations and reflections as well as in concrete pieces of work, what we have been calling “transmission of knowledge”, in its many facets, starting from its personal, intellectual and professional impact on us and our work.

As part of this process, in the spring of 1997 Laura Tedeschini-Lalli invited me to participate with a short cycle of conferences in the mathematics course she was then teaching at the School of Architecture, University Roma 3. During the mini-course, whose material can be found in [4] and [5], I reflected on the evolution of the concept of space in mathematics and painting during the XIXth. and XXth. centuries. When trying to understand the mathematical

---

notions and tools developed along these centuries, I invited the students to, and provided the material for, keep in mind the art painted at each specific time as a graphical reference from where to select some of the characteristics of the abstraction process we were looking at in mathematics.

While preparing the project required for the course, some of the students asked Tedeschini-Lalli if it would be possible to work in the opposite, complementary, direction: to keep in mind the (abstract) tools being used by the mathematical community at a specific time while looking at artworks, in order to select some questions, and a method. This question lead Tedeschini-Lalli and her students to the analysis of Maya with Doll (see appendix). Although the possibility of working in the direction they follow had already emerged in many of the conversations with mathematicians and artists that have nurtured and given shape to this process along the years, the study of Maya with Doll was the first actual contribution in this direction, and the one that gave me the push to trust my pencil as a tool of deep thought outside mathematics.

INTRODUCTION

How we look, the glance with which we confront what is around us, conditions what we see, which in turn gives shape to a new way of looking. For example, our spectations on the basic structure a house should have, strongly affects how we look at the houses we find when we first travel through countries and cultures different from ours. In turn, the diverse aspects found in buildings in a foreign place makes us look differently at our own houses, and often improve them, once we return home.

Fermat’s Last Theorem is perhaps the most famous problem in the history of mathematics. Its history, as well as an explanation of how Andrew Wiles finally solved it in 1994 by combining local and global techniques, can be found in [11], [12], [22], [25], [30] and [31] in the bibliography.
And this glance that each of us carries, is not a given a priori in the individual: our eyes are culturally trained since birth. The glance is culturally shaped. By its guiding us in the selection of characteristics when we look at an object, culture functions as a pair of glasses we always carry on. To become conscious of the fact that we are wearing a specific type of glasses allows us to change them at will, for example when the lenses have become obsolete and prevent us from seeing clearly, or when we want to experiment what happens when we look differently.

All disciplines concerned with understanding, explaining or representing what there is around us contribute to this shaping of the glance. Specially so, mathematics. Mathematics classes train us from childhood to look in a certain way, to develop the kind of lenses behind which later on, as adults, we will look at the world around us. To choose the aspects that specifically characterize a certain type of objects, to relate different shapes and structures, to classifying the patterns of a behaviour or to describe with precision the analogies among diverse things and phenomena, are all activities that take place while we look, and they are precisely the activities we learn from the mathematics we are taught at school. It is thus not surprising that in most disciplines and in most periods we can find traces of the contemporary mathematical glance.

Since the work of painters is done from the eyes and for the eyes, painting has so far given one of the most direct and explicit testimonies of how we look at a given moment. For this reason, it is through the work of painters that we have chosen to describe some of the problems that mathematicians -and occidental culture in general- has faced along the XX century.
As mathematicians, we will look at paintings with the same glance we carry when we look at mathematics. We will ask, when in front of a painting, the same questions we ask when in front of a piece of mathematics. In doing so, we will make explicit some problems that have concerned nineteenth and twentieth century occidental culture as shared by both painters and mathematicians, among others. This will provide the reader with some understanding of the way mathematicians look, as well as some new, and hopefully pleasurable ways, of looking at paintings.

Going into the exact relation between science, specifically mathematics, and art, specifically visual art, though fascinating, is a difficult and touchy one, with many social and political implications; it is neither easy nor trivial, nor can be done quickly in a few lines. Fortunately, for our purpose along these pages, the question of whether science and art are very different or very similar, or whether mathematicians and artists are aware of the possible connections between their respective works, is irrelevant. Thus we choose to leave the subject aside and not address it, refering the reader interested in it to the excellent articles by the historians of, respectively, science and mathematics, Thomas Kuhn and Catherine Goldstein listed in the bibliography ([10], [17]).

2. A DESCRIPTION OF A MATHEMATICAL WAY OF LOOKING.

As has been explained before, our aim is to place ourselves in front of a few paintings wearing a certain type of mathematical glasses (there are many different lenses through which mathematicians look), and describe what we see through them. We will start our project with a brief description of such glasses, and a sketch of part of the process that took mathematicians to put them on.
In their search of descriptions of what there is around us, mathematicians have followed through centuries a path which has constantly make them move from looking locally to looking globally. A process we could think of as the zooming in and zooming out of a camera. Although at each specific moment both ways of looking appear combined, the challenge at each moment seems to be in one direction or the other. In this sense and to make our frame of discussion explicit, we will risk over-simplification and divide the process so far covered in three periods.

1. From the local glance of Antiquity to the global glance of the XVIII century.
2. From the global glance of the XVIII century to the local glance of the beginning of the twentieth century.
3. The passage from local to global during the XX century.

Period 1: From the local glance of Antiquity to the global glance of the XVIII century.
(From the first mathematics of, say, Mesopotamia, Egypt, India and Greece, to the mathematics we find in Europe by the end of the XVIII century. (For a detailed description of this period see, for example, [20] and [21] in the bibliography.)

This first period takes us from the geometry of the individual objects of Antiquity up to the XVIII century, when for the first time Space appears explicitly described in mathematics as an immense cubical container, a huge shoe box. During centuries, mathematics were made on concrete objects. Lines, planes, circles, angles, etc., for example, were considered individually, with out reference to any frame or ambience space. Slowly, mathematicians relised that in their works and glances they were implicitly assuming that the geometric objects lived in an ambient or space with very specific properties: infinite, three-dimensional, homogeneous and offering no resistance to motion. This first mathematical space with its analytic model of cartesian and polar coordinates, suggested by Newton and described with all precision by Euler in his
Introductio, is known today under the name of Euclidean Space. Descriptions of how Euclidean Space came to be, can be found in [11] and [12] in the bibliography.

Period 2. From the global glance of the XVIII century to the local glance of the beginning of the XX century.

At the end of the XVIII century mathematicians identified space with Physical Space, the space where physical phenomena take place. And from a corner in this space, Euclidean Space, conceived as a huge box in which the different objects float, mathematicians observed, described and constructed. But concrete problems such as the making of maps of the Earth kept forcing mathematicians to leave behind their corner in the Euclidean box and get close to the surface of the objects, touching them (a more detailed description of this process can be found in [19] in the bibliography).

In 1818 Carl Friedrich Gauss, director of the astronomical observatory of Göttingen since 1807, carried on a detailed geodesic study of all the land covered by the kingdom of Hannover. The standard procedure to carry on a geodesic study is called triangulation: strategic places are marked on the land we want to study, and then the distance from each point to all of the rest is carefully measured, covering this way the region with a net of triangles whose sides and angles have been determined as precisely as possible. Notice that since the sides of the triangles are drawn directly on the Earth, which is not flat, these sides will not be straight, nor will the corresponding triangles be ordinary triangles. They will be what we call geodesic triangles: triangles directly marked on a surface (in this case the surface of the Earth). Thus, one of the first problems Gauss had to face was the real shape of the Earth, flattened (about 20 kms) at the poles and bulging at the equator, and hence closer to an ellipsoidal shape than to a sphere. At first thought it seemed to him—and to every body else involved in map making—that a detailed
knowledge of the exact shape of the Earth was necessary to correctly interpret the data obtained through the measurements. Gauss' greatest contribution was to turn this problem around, and instead of studying how the shape of the Earth affects the data obtained in a geodesic study, he determined how a geodesic study could be used to deduce the shape of the Earth.

Up to Gauss, the shape of the Earth had been deduced from the study of the Sun and the stars. Gauss proved that this was not necessary, that in order to find out the shape of the Earth it would suffice to carry on geodesic measurements on its surface. Furthermore, Gauss proved that it is not necessary to step out of a surface -like observing the Sun and stars in the case of the Earth- to determine its shape. This is exactly what the expression intrinsic geometry of a surface means: the geometry -shape- of a surface not only characterizes it, but it can also be described from the surface itself, without leaving it.

As an example, it is a fact that in flat geometry the angles of any triangle add up to 180°. For geodesical triangles on the surface of a sphere, the sum of the angles always adds up to more than 180°, while on a surface shaped like a saddle the angles of a triangle will add up to less than 180°. Now, let us suppose we want to determine the shape of a surface. We can triangulate it and measure the size of the angles in each triangle. Where all of the geodesic triangles have angles which add up to 180°, we know our surface must have a flat or a cylindrical shape; in the regions of the surface where the triangles' angles add up to less than 180°, the shape will be similar to that of a saddle; and where the angles of the triangle add up to more than 180°, we will have a shape similar to a piece of sphere or ellipse.

Getting close to the objects forced mathematicians to change their glance: as soon as we place ourselves on the surface of a sphere, for example, we find ourselves in a non-Euclidean world. How can non-Euclidean worlds exist within an Euclidean Space? By the middle of the
nineteenth century mathematicians had realised that the identification between Physical Space and mathematical space was only a convention, and a very limiting one, and lead by Riemann ([23]), they started to leave it behind. Once they gave themselves permission to abandon such restriction, they were able to understand (in a slow process that took from the works of Riemann of 1854 to those of Hausdorff in 1914) that any relation between arbitrary objects can in fact be used to construct a spatial structure. In 1883 Ascoli ([1]) published works in which curves were treated as points of a geometrical space, Volterra did the same thing with functions in 1887 ([28], [29]), and by 1914 Hausdorff reached a definition of mathematical space more adequate and less restricting that the cubical container: any net or web of relations between objects ([15]). This jump in the conception of space is clearly illustrated by the paintings 
*Las Meninas* of Velázquez (1656) and *Las Meninas* of Picasso (1957) (figs. 1, 2).

**Figure 1:** Velázquez, *Las Meninas*, 1656 (Museo del Prado, Madrid).

**Figure 2:** Picasso, *Las Meninas*, 1957 (Museo Picasso, Barcelona).

For example, in Velázquez’s canvas the space between the princess and María Agustina Sarmiento, the maid kneeling in front of her, is a cubical container external to both of them, the room in which the scene takes place. A room that, such as is represented in the painting, would not change if the girls were not in it. On the other hand, the space between these same figures in Picasso’s painting is a net formed by the visual and positional relations between them both.
The way in which each of them sees the other and the position in which each of them is placed with respect to the other, produce the triangular web that as spatial structure connects them among themselves and with the rest of the figures in the room. The scene as a unique global whole is the outcome of a multiplicity of local points of view and relations glued to each other by means of a spatial structure consisting in triangles, rectangles and the other basic geometric figures used by Picasso. If we were to remove the girls from the room in Velázquez’s representation, nothing would have to be changed. If we were to do the same thing in Picasso’s painting, the whole thing would have to be painted again.

**Period 3: The passage from local to global in mathematics during the XX century.**

In 1914 Hausdorff defines a mathematical space as formed by any set of objects and the relations among these objects. After the work of Hausdorff, whenever we have a relation established among the elements and subsets of a set, we can use it to build a space. The specific properties of the relation defined will characterize the spatial structure thus obtained. Different relations will have different properties and will produce different spatial structures. There is no more “Space”; there are “spaces”, as many as sets of objects and relations among these objects. These spaces are no more necessarily global containers, like the space of Newton, but web spaces defined locally by means of neighbourhoods.

Several problems emerge when we work locally. One of them is that on a local view we cannot grasp an object in a unique description. If we look at a face from far away in the distance, we can cover all of its features in one glance. But if we get very close to it, we can only see at most one feature at a time. In order to see all of them we must move our eyes, obtaining a series of local views which we must then glue together in order to reconstruct the whole face. This brings up a second problem: how to glue all the local pieces of information in
a coherent way. If we had never seen a face before, we would not know where to place the eyes in relation to the nose or mouth unless we had a map of the underlying structure common to all faces: eyes above, nose in the center, the mouth below the nose and under the nose the chin, for example.

The construction of these structures underlying the things around us, requires a process of abstraction: to distinguish between what is particular (this face) and what is general (faces). These structures allow us, among other things, to distinguish objects of different types, to classify and recognize objects of the same type and to glue local data in a coherent way.

An example of a first type mathematical structure is the structure of the set of symmetries in a pattern. How many different ways are there of constructing a wall paper?, asked themselves some mathematicians. How many different ways are there of covering a wall with tiles?, equivalently asked themselves some other mathematicians. George Pólya proved in 1924, that there are seventeen ways of doing it, all of which can be found at the walls of Alhambra, the arab palace in Granada.

![Figure 3: The seventeen types of symmetries of a wall paper, after George Pólya in “über die Analogie der Kristallsymmetrie in der Ebene”, Zeitschrift für Kristallographie, 1924.](image-url)
An example of a second type of mathematical structure is that of the set of points with rational coordinates on an elliptic curve. The number 210 can be written both as $14 \times 15$ and as $5 \times 6 \times 7$. How many numbers are there with this property of being simultaneously the product of two and three consecutive numbers? Said differently, how many numbers are there so that we can find two other numbers $x$ and $y$ with $N = y(y+1)$ and $N = (x-1)x(x+1)$? Looking for an answer to this question is equivalent to searching for all the points $(x,y)$ with integer coordinates on the curve $y^2 + y = x^3 - x$, a curve of the type called elliptic. These curves have the unusual property that the points on them with rational coordinates (which obviously contain the points with rational coordinates) have a surprising structure formed by lines: if we join any two of them by a line, this line will cut the curve in a third point also with rational coordinates (for an introduction to the subject of elliptic curves see [18]).

![Figure 4](image)

**Figure 4.** The structure of the set of points with rational coordinates on the curve $y^2 + y = x^3 - x$ (after Tate and Silverman, *The arithmetic of elliptic curves*, Springer Verlag, :, UTM 1992).

Structures like these two we have just seen are commonly used by mathematicians, among other things, as underlying scaffolds or frames on which to put together the partial pieces of information they obtain when they work locally. They serve as basic skeletons that guarantee that the ensemblage of local data is done coherently, so useful global information can be obtained.
3. A MATHEMATICIAN’S LOOK AT SOME PAINTINGS OF PICASSO

In the preceding section we described a certain type of lenses or methodology that mathematicians have developed along the last two centuries. It is a combination of local and global glances at the objects under their studies. As a first step, and starting with an abstract glance, they choose some general characteristics that provide, so to speak, a global sketch of the object under study. Looking then locally, they obtain a series of more detailed informations about partial aspects of the object. The third step consists in weaving this local data into the initial abstract sketch, obtaining as a result a collage-type of description that provides much information, some of it available in the global general structure of the object seen as a whole, some codified in the local pieces that conform this whole. As we will see in this section, as well as in the following appendix, this same methodology has successfully been used by some XX century painters.

From 1950 to 1957, Pablo Picasso carried on a series of paintings and studies on the scene represented by Velázquez in his 1656 piece “Las Meninas” (fig. 1). We already saw one of these paintings, it is our figure 2. Let us now look at another of the canvases in this series.

Figure 5. Picasso, Las Meninas: María Agustina Sarmiento 3, 1957 (Museo Picasso, Barcelona).
At a first glance, this is a strange painting. The underlying global structure (eyes above, nose in the center, the mouth below the nose and under the nose the chin) allows us to recognize a person: her face and her torso. But it is not a face as we would get, for example, from a photograph. In a photograph all features of a face match with each other. In contrast, in this painting features do not seem to match with each other. They are combined in such a way that we still recognize the face as a whole, but they definitely do not match with each other.

If this painting is not a photographic portrait, what is it then? What is Picasso exactly describing? By the title that Picasso gave it, we know that this painting is a reflection on Velázquez’s *Las Meninas* (fig. 1), and thus we may assume that it is telling us something about Velázquez’s piece. But, what exactly? In the following paragraphs I will search for an answer to this question, and to do so I will carry on the glance developed by Gauss at the beginning of the eighteenth century -thus placing myself equidistant in time from both painters-, and on the surface of things. As Gauss did, I will look at the local information intrinsic on Picasso’s painting, and see what it is telling us about the work of Velázquez. Said differently, I will pay individual attention to the pieces of local information that conform the image of Picasso. I will isolate some of the partial images that he so brilliantly assembled, and try to decodify the information about Velázquez’s painting hidden in each of them. I invite the reader to follow me in this experiment.

I start from the beginning, looking at Picasso’s canvas globally, as a whole. It reproduces the face and torso of a lady, María Agustina Sarmiento. Who is María Agustina Sarmiento? She is the maid who, placed in the center of the scene painted by Velázquez, gracefully offers water to the young princess. Since I learnt about the habit of eating clay jars fashionable among the ladies in the court of Spain at the time of Velázquez ([24]), the little clay pot is the first thing that my eyes look for whenever I see a representation of María Agustina Sarmiento. Once more,
I am moved by Picasso’s talent: he only needs a couple of black lines to describe the jar on the tray. Just a few black lines. Thinking of black lines, those describing the tray look peculiar. There is something in them that strikes me as odd, but I cannot identify right away what it is exactly.

Mathematicians work with pen and paper. I open my notebook and start drawing the tray as I would draw it if I were Picasso-spectator looking at Velázquez’s painting. I compare trays, and realise that in my drawing the black lines indicating the interior shape of the tray appear, from the spectator’s point of view, behind the jar, not before it as they do in Picasso’s canvas. Could it be that Picasso’s lines are drawn as seen by the painter Velázquez, not by the painter Picasso looking at the painting of Velázquez? I study again Picasso’s tray, and reproduce in my notebook its structural lines (fig. 6).

![Illustration 6: Sketch of the tray in Picasso’s painting María Agustina Sarmiento.](image-url)

I go back to the painting of Velázquez (fig. 1), and conclude that if Picasso’s black lines in the inside of the tray were painted from Velázquez’s point of view, they would appear under the jar’s belly, not under its handle. From where in Velázquez’s room are these lines seen as Picasso has painted them? I let my mind move around the scene, and as I do so I draw in my notebook the tray as I see it from the different places. When I am done I compare my drawings...
with the tray in Picasso’s canvas and the conclusion is immediate: it is María Agustina herself who sees the tray as Picasso describes it.

Is then Picasso’s painting a painting of María Agustina as seen by María Agustina? A few seconds suffice for me to realise how absurd this suggestion is: the tray is described as seen by María Agustina, but the young lady cannot see her own eyes, which in fact, as Picasso has painted them, do not match with each other. Not only none of the eyes is seen by María Agustina, but each of them is seen from a different place.

New working hypothesis: Picasso abandons the fixed point, and physically moves around the figure of María Agustina, chosing a few places from which he gives us local descriptions of the woman, which he then assembles together in a unique global description. Before I proceed to check the validity of this new hypothesis, I reflect on it. Does it seem absurd? No, in fact it makes sense. After all, if it turns out to be correct, it would just be a twentieth century version of the same game played by Velázquez: to move from his eyes to those of the king, queen and spectators, and from these, through the mirror, to those of the visitor who appears at the door in the back of the room. As I put into words what Velázquez did, a switch is turned on in my mind: eyes, that is what this first clue –the tray- is telling me, eyes. In Velázquez’s painting, different objects and figures are seen from different eyes. Wouldn’t it make all the sense, then, that Picasso would try to describe one single figure as simultaneously seen by different eyes? Not only I am now convinced that my new hypothesis is not an absurd one, but I can even state it in a precise way: Picasso is pushing Velázquez’s game a step further, and simultaneously describes María Agustina as seen by some of the different eyes present in the room of Velázquez.
Let us test this new hypothesis. The next clues I need must be in hidden in the remaining local data: in the hand holding the tray (fig. 7), in the frame of the face (fig. 8) and in the features of the face (figs. 9 and 10).

Figure 7: Sketch of the hand holding the tray in Picasso’s painting *María Agustina Sarmiento*.

While drawing the hand which holds the tray, I become aware of a detail I had so far missed: in this hand the inner lines of the thumb and the palm are carefully marked. I soon realise this is the clue I am searching for. From where in Velázquez’s scene can these lines be perceived? I study Velázquez’s painting (fig. 1). One could only perceive the inner lines in the girl’s thumb and palm if standing to her left, in fact exactly where Velázquez himself stands.

The tray as seen by María Agustina, the inner lines of palm and thumb as seen by Velázquez, … I have already found traces of the eyes of two of the persons present in the room. The search is exciting, and I go on drawing, in pursue of my next clue.

Figure 8: Sketch of the frame of the head in Picasso’s painting *María Agustina Sarmiento*. 
On reproducing them, I notice that both hair and face are completely outlined, the hair framing the face perfectly, as can only be seen by the princess and her attendants, standing exactly in front of María Agustina in Velázquez’s scene.

María Agustina herself, Velázquez, the princess and her attendants… Yes, it seems as if Picasso has successively placed himself exactly where these people stand in Velázquez’s description and, trained in the process of abstraction, has chosen some characteristic of the figure of María Agustina to draw as seen from each of those standpoints. In this way, the presence and eyes of this people can be reconstructed from the local information codified in Picasso’s painting. María Agustina Sarmiento describes to us the tray where the jar stands, Velázquez the hand holding the tray, the princess and her attendants the frame of María Agustina’s hair and face.

A bold and daring question comes to mind at this point: Could it be that Picasso has managed to leave in this figure traces not of some of the eyes, but of all of the eyes present in Velázquez painting? If this were the case, we would have found the answer to our initial question: Picasso’s painting is not a portrait of María Agustina, but a collage of local views of María Agustina which codifies the information of where in the room did Velázquez place his characters by describing where they stand with respect to María Agustina, and hence to each other. By looking at Picasso’s painting we don’t know the exact number of people in Velázquez’s painting, but we know from how many angles is María Agustina looked at, that is, where in Velázquez’s room there is people. In this form, Picasso’s portrait of María Agustina would codify a precise description of the space in Velázquez room, since it would hide all the clues necessary to reproduce the web of positional or spatial relations among the characters in Velázquez’s scene.
But I am going too fast, I must finish checking my hypothesis before I jump to conclusions. I try not to think on anything outside what I am seeing, and I start drawing the left eye.

![Figure 9: Sketch of the left eye in Picasso’s painting *María Agustina Sarmiento.*](image)

On drawing it, I find the three essential clues codified in this eye: the horizontal black line on the left, the accumulation of grey on the right, and the position of the pupil. These clues indicate that the eye is drawn from the girl’s right profile, that is, the place occupied by the queen, king and spectators in Velázquez’s painting. The same place, I notice, from where the jar is seen. This makes sense, the parents are worried: has their daughter already been initiated in the habit of eating clay jars?

I place reproductions of both paintings in front of me once more, looking from one to the other: Picasso has already described the figure of the girl with the jar from above, from the right, from the left and from the front of Velázquez’s scene. He has so far offered us the point of view of the girl herself, of Velázquez, of the king, queen and spectators, of the princess and the attendants to her left. Is there anything missing? Has everyone already spoken his or her view? What about the back? Is there anyone standing in the back of the room? Yes, there is a man appearing through a door. Does it make sense to search for traces of his eyes in Picasso’s canvas? Is it possible that Picasso has described María Agustina as seen from above, from the
right, from the left, from the front and from the back of the room, that is, from all fourth sides, plus from above? Can a figure be described in a painting from all those local points of view simultaneously, that is, from all around and in a global image, without the resulting collage dissolving into an absurd and irrecognizable mess? Two reasons lead me to trust in Picasso’s success: first, that it is possible to do it in mathematics – Projective Space, for example, as a collage of all the affine views-, and second, the enormous talent of this painter. If it can be done, he will do it. There is a feature missing in my analysis: the right hand side eye. I draw it, and on doing so I notice it is seen from the same place as the nose.

**Figure 10:** Sketch of the right eye and nose in Picasso’s painting *María Agustina Sarmiento*

I study once more Velázquez’s scene. Where is this profile seen from? From the left of Sarmiento, that is, from the back of the room. It is, thus, an eye described as seen by the man standing at the door and by the image of the king and queen in the mirror: María Agustina seen from the back of the room! The missing fourth side!

A person described from the front, from behind, from the left, from the right and even from her own eyes, from above. Assembling together all these local and partial views, Picasso offers us a complete global view. If one tries to describe a volume on a flat surface by patching together partial views, one can not do it in a more complete way: Picasso covers the three axis. In the axis which indicates depth he places us in front and behind María Agustina. Along the
width axis, he takes us to her left and to her right. In the third axis, in the height dimension, he places us over MaríaAgustina´s hand, at her eyes level.

The analysis of Maria Agustina Sarmiento offers an example of how XX century painters, proceeding as their contemporary mathematicians, have managed to reconstruct the dissembled reality around them, not by creating with tricks –color grading, for example– a fictitious uniform globality, but by carefully glueing together the local views into a coherent global collage.

The reconstruction of a global view by assembling several partial points of view is a very fructiferous methodology, not only in mathematics and painting, but in life in general. It is very common, for example, that a situation that takes place within a group of people is felt and described as a totally different scene by each of the persons involved in it. Only by putting together the partial descriptions offered by the different viewers can one attempt to know which a fair degree of accuracy what actually took place. The next painting of Picasso, Femme assise, offers an example of how the game local/global can be played in order to weave in a unique tale two different stories.

Figure 11: Picasso, *Femme assise*, 1971 (Private collection, Spain). (To be reproduced in color)
The first time I saw this painting, for the longest time I could only see a pair of huge blue and green eyes, like those of a bird, superimposed on a pale pink feminine sex. On closer analysis, I realised that, in fact, the enormous figure depicted in the canvas is in fact divided in two clearly distinct parts, and that with each of this parts Picasso is telling us a story. Once again, I invite the reader to follow me in my search of the local clues that provide the information codified by Picasso.

The upper half of the canvas is painted in blue and green, the lower half in pink and white. What is there, exactly, painted in blue and green, what in pink and white? In green and blue I see two eyes, a nose, a mouth, and a thumb (to the left of the other fingers). In white and pink there is a naked foot, a hand, a female’s sex, part of a breast and the right hand side of a line of hair. Everything, no matter the color, as seen by an observer placed in front of it. Title of the painting: *Femme assise*, woman seated. I can now clearly see her, a woman naked and sitting, painted from head to toes: the pink and white part of the painting. But, what about the rest of the painting? What about those eyes and that thumb?

Those are not her eyes, they are the eyes of someone staring at her while holding up a thumb. Who would look at someone, concretely at a naked woman, while holding up a thumb? A painter, of course! The thumb is the clue. When trying to reproduce in a drawing a figure standing in front of me, how do I measure its proportions? With my thumb! I look at the painting again; here they are, the painter and his woman model. The painter that looks at a woman and meassures her proportions with his thumb. Is this, then, all that Picasso is describing, a painter and his model? I don’t think so. I have seen several dozens of paintings in which Picasso drew a painter and his model, and in none of them have I seen this game of superimposition. No, he must be trying to describe something else than a painter man and a model woman. Which is the object represented in this painting?
In the canvas of María Agustina Sarmiento (fig. 6), Picasso codified the spatial relations among the persons present in Velázquez’s scene, and in doing so he managed to give description of a volumen on the flat surface of a canvas by glueing together local and flat views of the object as perceived from all possible sides; front, back, left, right and above. What is Picasso’s aim in this new painting? If he had wanted to represent only a painter while painting a model, he would have done so, as he did in many other of his paintings and engravings. But if it is not a just painter and his model, what is it, then?

I go back to the painting and focus my attention on the man. He is looking at the woman’s body, at her sex. In fact he is devouring her with his eyes. Next, I focus on her. She is looking at him looking at her body. This is a description of what happened between the painter and the model, simultaneously told by those two who lived it: the painter and the model. How each one was seeing the other, the effect that each one of them had on the other. And in order to tell simultaneously both stories, Picasso superimposes both views, managing to describe the space between them through the relation that is established among them.

**Conclusion**

During the XVIIth century, the space between the painter and his models was the mathematical space of Newton and Euler, the global ethereal cubic container that we see represented as a room in *Las Meninas* of Velázquez. By the XXth century, as the paintings of Picasso we just looked at admirably illustrate, the space between the painter and his models became, like the mathematical space of Hausdorff, a byproduct of the local relations among them. This new conception of space has make it possible, for both mathematicians and painters, to combine local and global tools in order to obtain a more complete description of the objects under their study.
This combination of local and global tools is not exclusive of mathematics or painting. In fact, during the twentieth century it became one of the essential abstract features of the occidental culture. We hope these pages will serve the readers as an illustration of how the mathematics we all studied at school can help us develop an individual glance with which to freely confront what is around us, a painting of Picasso, for example. Mathematics train us to ask our own questions, to create our own methods, and to recognize the abstract lenses through which our culture looks.

It is clear that different representations of the same object clutter or enhance some of its characteristics; a choice of characteristics implies knowingly choosing some and altogether disregarding others. This mental representation, in turn, shapes the expectation when actually looking at objects, because it shapes the selection of characteristics to be observed: the glance is culturally shaped. The assumptions and choices implicit in our representation of phenomena or weaved into our abstract modelling are sometimes recognized as the obstacles when people of different background communicate. Imagining an object forces and shapes a mental representation of it, and this abstract level of description goes usually undiscussed. (Laura Tedeschini-Lalli in Look around as you go: abstract glasses, concrete crossroads. [4] pp. 9-11).

BIBLIOGRAPHY:


