BISHOP-PHELPS-BOLOBÁS THEOREM, INTEGRATION OF MULTI-FUNCTIONS AND BOUNDARIES

BERNARDO CASCALES
UNIVERSIDAD DE MURCIA

ABSTRACT. The common link for the three items above is fragmentability. We will briefly present how we have used fragmentability over the past years to obtain some new results connected with the last two items in the title. Then we will discuss in detail a strengthening of the Bishop-Phelps property for operators that in the literature is called the Bishop-Phelps-Bollobás property. Let X be a Banach space and L a locally compact Hausdorff space. We have proved recently that if $T : X \to C_0(L)$ is an Asplund operator and $\|T(x_0)\| \approx \|T\|$ for some $\|x_0\| = 1$, then there is a norm attaining Asplund operator $S : X \to C_0(L)$ and $\|u_0\| = 1$ with $\|S(u_0)\| = \|S\| = \|T\|$ such that $u_0 \approx x_0$ and $S \approx T$. As particular cases we obtain: (A) if T is weakly compact, then S can also be taken being weakly compact; (B) if X is Asplund (for instance, $X = c_0$), the pair $(X, C_0(L))$ has the Bishop-Phelps-Bollobs property for all L; (C) if L is scattered, the pair $(X, C_0(L))$ has the Bishop-Phelps-Bollobs property for all Banach spaces X.