THE DUGAVET PROPERTY AND WEAK NEIGHBORHOODS IN BANACH LATTICES

ANNA KAMIŃSKA
UNIVERSITY OF MEMPHIS, USA

ABSTRACT. The Daugavet property and the diameters of relatively weakly open subsets of unit balls in Banach lattices X on measure spaces are studied. It is shown that under mild assumptions the subspace X_a of order continuous elements inherits the Daugavet property from X. This is applied to prove that if X has the Daugavet property and the Köthe dual X' is strictly monotone (resp. order continuous) then X' contains a lattice isometric (resp. isomorphic) copy of $L_1(0,1)$. These results yield that a large class of r.i. spaces including several interpolation sums fail the Daugavet property and also that any r.i. space over a finite atomless measure space with the Daugavet property coincide to either L1 or L8. Applications are shown for Orlicz, Lorentz, Marcinkiewicz spaces as well for Nakano spaces. It is established that in most cases these spaces do not enjoy the Daugavet property. However, it is proved that in a large class of Orlicz or Nakano spaces, in particular those induced by fast growing Orlicz functions, all relatively weakly open subsets of their unit balls have diameter two.

These results are joint with M.D. Acosta and M. Mastylo.