DUNFORD-PETTIS PROPERTIES FOR LOCALLY CONVEX SPACES

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Abstract. Let E be a (real or complex) locally convex space, E^* its dual and E^{**} its second dual space.

In [G1, Def. 1] E is called to have *the* D. -P. *property* if every continuous linear mapping of E into any locally convex Hausdorff space Y which maps bounded sets onto weakly relatively compact sets necessarily maps *absolutely convex weakly compact sets* onto relatively compact sets.

In [G1, Def. 2] E is called to have *the strict* D. -P. *property* if every continuous linear mapping of E into any locally convex Hausdorff space Y which maps bounded sets onto weakly relatively compact sets necessarily maps *weakly Cauchy sequences* onto Cauchy sequences.

In [G2, Ch. 5, P. 4.2, Ex. 2] E is called to be a DP *space* if every continuous linear mapping of E into any locally convex Hausdorff space Y which maps bounded sets onto weakly relatively compact sets necessarily maps *weakly compact sets* onto relatively compact sets.

We say that a locally convex space E has *the sequential Dunford-Pettis property*, if for every weakly null sequence (x_n) in E and for every $\sigma(E^*, E^{**})$-null sequence (x_n^*) in E^* we have that $\lim_n x_n^*(x_n) = 0$. It is well-known that a Banach space E is a DP space if and only if it possesses the sequential Dunford-Pettis property (see, e.g., [B], [D]).

We will discuss the next statement which seems not to be so well-known.

Theorem. (a) ([BL, p. 397]), [T, Theorem 4. 13]) A metrizable locally convex space E is a DP space if and only if it possesses the sequential Dunford-Pettis property.

(b) In general, a locally convex space E may have the sequential Dunford-Pettis property without being a DP space.

(c) In general, a locally convex space E may be a DP space without having the sequential Dunford-Pettis property.

(d) In general, a normed space E may have the D.-P. property, without being a DP space.

This talk (as well as [T]) is iniciated by [MT], where an analogue of the sequential Dunford-Pettis propery in the general framework of topological Abelian groups is introduced and studied.

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