# ON $\psi$-DIRECT SUMS OF BANACH SPACES

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ABSTRACT. Based on a fact in Bonsall and Duncan [1] (see [3]) concerning the correspondence between an absolute between an absolute normalized norm on $\mathbb{C}^2$ and a convex function on the unit interval with some conditions, we introduced in 2002 the notion of $\psi$-direct sum $X \oplus_\psi Y$ of Banach spaces $X$ and $Y$ ([5]; see [2,4] for finitely many Banach spaces). Since then it has been attracting a good deal of attention and many investigations have been done. In this talk some of the recent results will be presented.

Let $\psi$ be a convex (continuous) function on the unit interval satisfying the conditions

(1) $\psi(0) = \psi(1) = 1$ and $\max\{1-t, t\} \leq \psi(t) \leq 1$ for all $0 \leq t \leq 1$.

The $\psi - direct\, sum\, X \oplus_\psi Y$ of Banach spaces $X$ and $Y$ is the direct sum $X \oplus_\psi Y$ equipped with the norm

(2) $\|(x,y)\|_\psi = \begin{cases} (\|x\| + \|y\|)\psi\left(\frac{\|y\|}{\|x\|+\|y\|}\right) & if (x,y) \neq (0,0), \\ 0 & if (x,y) = (0,0) \end{cases}$

([5]; see [2] for finitely many Banach spaces). This extends the notion of $l_p$-sum $X \oplus_p Y$ and the $l_1$- and $l_\infty$-norms are the largest and the smallest such norms respectively: $\|.\|_\infty \leq \|.\|_\psi \leq \|.\|_1$.

Our concerns is how various (specially geometric) properties of $X \oplus_p Y$ are described by means of those of $X$ and $Y$, and $\psi$.

**References** [1] F. F. Bonsall and J. Duncan, Numerical Ranges II, London Math. Soc. Lecture Note Ser. 10 (1973). [2] M. Kato, K.-S. Saito and T. Tamura, On the $\psi$-direct sums of Banach spaces and convexity, J. Aust. Math. Soc. 75 (2003), 413-422. [3] K.-S. Saito, M. Kato and Y. Takahashi, Von Neumann-Jordan constant of absolute normalized norms on $\mathbb{C}^2$, J. Math. Anal. Appl. 244 (2000), 515-532. [4] K.-S. Saito, M. Kato and Y. Takahashi, On absolute norms on $\mathbb{C}^2$, J. Math. Anal. Appl. 252 (2000), 879-905. [5] Y. Takahashi, M. Kato and K.-S. Saito, Strict convexity of absolute norms on $\mathbb{C}^2$ and direct sums of Banach spaces, J. Inequal. Appl. 7 (2002), 179-186.