SOME RATHER WEIRD NORMS ON $\mathbb{R}^2$

BOGDAN C. GRECU
QUEEN'S UNIVERSITY BELFAST

ABSTRACT. The polynomial numerical indices of a Banach space are constans, denoted by $n^{(k)}(X)$ relating the norm and the numerical radius of homogeneous polynomials on the space. This concept is a generalization of the *numerical index* of a Banach space, $n(X)$.
If $X = (\mathbb{R}^2, \|.\|)$ and $n(X) = 0$ then it can be shown that $\|.\|$ must be the Hilbertian norm. The situation for numerical indices of higher order is not so tidy. For instance, $n^{(p-1)}(l_p^2) = 0$ if p is an even number and, actually, $n^{(2k-1)}(X) = 0$ if $(X, \|.\|)$ is a real Banach space of dimension greater than one such that the mapping $x \longrightarrow \|x\|^{2k}$ is a $2k$-homogeneous polynomial.
The following results hold

(1) If $k_0 = min\{k : n^{(k)}(X) = 0\}$ then k_0 is odd.
(2) If $k_0 = 3$ and $\|.\|$ is a normalized absolute symmetric norm then its fourth power is a polynomial.
(3) If $k_0 = 3$ is not a symmetric norm or if $k_0 \geq 5$ then the $\|.\|$ could be a norm (a "weighted product" of l_p-like norms) whose no power is a polynomial.

We will give a description of these rather weird norms.
Joint work with D. García, M. MAestre, M. Martin and J. Meri.