APPLICATION OF AN IDEA OF VORONOI TO LATTICE PACKING AND ZETA FUNCTIONS

PETER GRUBER
INSTITUT FUR DISKRETE MATHEMATIK UND GEOMETRIE (AUSTRIA)

ABSTRACT. Apositive definite quadratic form on E^d can be identified with its coefficient matrix or its coefficient vector and thus with a point in $E^{d(d+1)/2}$. Then certain problems for forms, ellipsoids, John type results, lattice packings of balls and zeta functions for lattices are translated into geometric problems in $E^{d(d+1)/2}$ which, sometimes, are transparent and easier to solve.
This idea of Voronoi is used to characterize refined extremum properties of the density of lattice packings of Balls and, similarly, for the Epstein zeta function. Extensions to lattice packings of smooth convex bodies, more general zeta functions and kissing numbers are discussed.
In the introduction an overview of the geometry of numbers are given.