CLASSIFYING MONOTONE COMPLETE C^*-ALGEBRAS

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Abstract. Each C^*-algebra has a self-adjoint part equipped with a natural partial ordering. A C^*-algebra is *monotone complete* if each norm bounded, upward directed set of self-adjoint elements has a least upper bound. For example, every von Newmann algebra is monotone complete, but the converse is false.
Let $B([0,1])$ be the algebra of all (complex) bounded, Borel measurable functions on the interval [0,1]. Let M be the ideal of all bounded Borel functions which vanish off some meagre set (depending on f). Then $B([0,1])/M$ is a commutative, monotone complete C^*-algebra which is not von Newmann. It can be embedded as a subalgebra of l^∞.
A unital C^*-algebra A is said to be *small* if there is a unital, complete isometry from A into $L(H)$, where H is separable. As small von Neumann algebras can be identified with von Neumann subalgebras of $L(H)$, where H is separable and infinite dimensional. Up to isomorphism, there are c small von Neumann algebras, where c is the cardinality of the continuum.
In joint work with K. Saito, we constructed a "weight" semigroup which classifies small monotone complete C^*-algebras into 2^c distinct types. We introduce an invariant for monotone complete C^*-algebras, the *spectroid*. It turns out that algebras which are classified by the same element of the semigroup always have the same spectroid.