A NEW NORM ON C^k-SMOOTH SPACES WITH PRI

ANTONIN PROCHAZKA
UNIVERSITÉ BORDEAUX 1

Abstract. For a large class of non-separable Banach spaces (the class to be specified in the talk) the following implication holds: If X admits a C^k-smooth norm (k natural or infinity), then X also admits a norm which is at the same time locally uniformly convex, C^1-smooth and which is approximated uniformly on bounded sets by C^k-smooth norms. The C^k-smoothness is understood in the Frechet sense. In particular, the conclusion of our theorem holds for the space $C[0, \omega_1]$, thus solving in a positive manner one problem in the Deville-Godefroy-Zizler book. We will start the exposition by exploring the context of this result (Asplund averaging, density of norms of higher smoothness, the separable and $c_0(\Gamma)$ predecessors of our result) and then we will have a closer look on the main ingredients of the proof.