SOBOLEV MAPPINGS INTO METRIC SPACES

JEREMY TYSON
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Abstract. The theory of real-valued Sobolev functions is by now classical and forms a cornerstone of the modern approach to PDE. Motivated by variational problems in differential geometry and geometric function theory in sub-Riemannian spaces and more general contexts, there is nowadays significant interest in extending the Sobolev theory to cover nonlinear targets such as Riemannian manifolds and even nonsmooth metric spaces.
This talk will review the history of Sobolev spaces of mappings into metric spaces. We will discuss recent advances connected with the density of smooth or Lipschitz mappings, absolute continuity and dimension distortion, and the existence of Sobolev surjections from Euclidean domains ('Sobolev Peano cubes'). Along the way we will encounter several striking results, including: Theorem: There exists a Sobolev map from R^2 to the first Heisenberg group H^1 which cannot be approximated in the Sobolev norm by Lipschitz mappings.
Theorem: Every locally compact and complete geodesic metric space is the image of R^2 by a continuous map in the Sobolev class $W^{1,2}$. I will explain why these results are surprising and indicate some of the techniques used in their proofs.