Rectifiable curves in Sierpiński carpets

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In the last years, there has been an intensive research on the setting of metric measure spaces, where a first order differential calculus has been developed. Standard assumptions in analysis on metric spaces include that the measure is doubling and the space supports a p-Poincaré inequality. In some sense, these conditions guarantee that any pair of points can be connected by a family of curves that are not too long and that the curves can be nicely distributed.

In this talk we study a particular case of doubling metric measure space. We focus our attention in a classical fractal: the Sierpiński carpet endowed with its associated Hausdorff measure.

In the first part of the talk, we will review some of the latest results which have contributed to understand the geometrical structure of metric measure spaces supporting a p-Poincaré inequality and explain why the families of curves that live in the Sierpiński carpet are not enough for our purposes; that is, in terms of Poincaré inequalities.

In the second part, we will characterize the slopes of nontrivial line segments contained in self-similar Sierpiński carpets. The set of slopes is related to Farey sequences and the dynamics of punctured square toral billiards. As a consequence, we deduce conclusions about the collection of everywhere differentiable curves contained in such carpets.

Joint work with J. A. Jaramillo (Universidad Complutense de Madrid), N. Shanmugalingam (University of Cincinnati), J. Tyson (University of Illinois at Urbana-Champaign), and A. Williams (Texas Tech University).