MORSE AND SARD MEET TAYLOR AND STEPANOV

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Abstract. Let n, m, k be positive integers with $k = n - m + 1$. We establish an abstract Morse-Sard-type theorem which allows us to deduce, on the one hand, a previous result of De Pascale's for Sobolev $W^{k,p}_{\text{loc}}(\mathbb{R}^n, \mathbb{R}^m)$ functions with $p > n$ and, on the other hand, also the following new result: if $f \in C^{k-1}(\mathbb{R}^n, \mathbb{R}^m)$ has a Taylor expansion of order k at almost every point $x \in \mathbb{R}^n$ and satisfies $|f(x+h) - f(x) - Df(x)(h) - ... - \frac{1}{(k-1)!}D^{k-1}f(x)(h^{k-1})| = O(|h|^k)$ for every $x \in \mathbb{R}^n$, then the set of critical values of f is Lebesgue-null in $\mathbb{R}^m$. In particular this allows us to establish the Morse-Sard property for several classes of functions with badly behaved derivatives of order $k-1$ (for instance, functions whose derivatives of order $k-1$ are pointwise Lipschitz, AKA Stepanov functions) to which none of the previous generalizations of the Morse-Sard theorem can be applied. This is joint work with Juan Ferrera and Javier Gómez-Gil.