TRANSLATION-INVARIANT SUBSPACES FOR WEIGHTED L^2 ON $\mathbb{R}_+$

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Abstract. In the setting of weighted L^2-spaces, a striking result due to Domar states that the lattice of closed invariant subspaces for $\{S_\tau\}_{\tau\geq 0}$ in $L^2(\mathbb{R}_+, w(t)dt)$ coincides with the lattice of *"standard invariant subspaces"*

$$L^2([a,\infty), w(t)dt) = \{f \in L^2(\mathbb{R}_+, w(t)dt) : f(t) = 0 \text{ a.e } 0 \leq t \leq a\}, (a \geq 0),$$

whenever w satisfies:

(1) w positive continuous decreasing in $\mathbb{R}_+$.

(2) $\log w$ is concave in $[c,\infty)$, for some $c \geq 0$.

(3) $\lim_{t\to\infty} \dfrac{-\log w(t)}{t} = \infty$ and $\lim_{t\to\infty} \dfrac{\log|\log w(t)| - \log t}{\sqrt{\log t}} = \infty.$

In this talk we present an extension of Domar's Theorem to a wider class of weight functions w not fulfilling condition (2), which is replaced by a geometric condition on $\{w(t_n)\}_{n\geq 1}$ for some strictly increasing sequence $\{t_n\}_{n\geq 1} \subset \mathbb{R}_+$ with the uniformly bounded condition $\sup_n(t_{n+1} - t_n) < \infty$. This extension addresses, in some sense, a question posed by Domar.

In addition, we provide an example of a weight function $\tilde{w}$ for which the lattice of closed invariant subspaces for $\{S_\tau\}_{\tau\geq 0}$ in $L^2(\mathbb{R}_+, \tilde{w}(t)dt)$ is non-standard such that $\tilde{w}$ satisfies (3) and the geometric condition is fulfilled only for sequences $\{t_n\}_{\geq 1} \subset \mathbb{R}_+$ with $\sup_n(t_{n+1} - t_n) = \infty$.

This is a joint work with Eva A. Gallardo-Gutíerrez (Universidad Complutense de Madrid, Spain) and Jonathan R. Partington (University of Leeds, UK).