ON THE SET OF LIMIT POINTS OF CONDITIONALLY CONVERGENT SERIES

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Abstract. Let $\sum_{n=1}^{\infty} x_n$ be a conditionally convergent series in a Banach space and let τ be a permutation of natural numbers. We study the set $LIM(\sum_{n=1}^{\infty} x_{\tau(n)})$ of all limit points of a sequence $(\sum_{n=1}^{p} x_{\tau(n)})_{p=1}^{\infty}$ of partial sums of a rearranged series $\sum_{n=1}^{\infty} x_{\tau(n)}$. We give full characterization of limit sets in finite dimensional spaces. Namely, a limit set in $\mathbb{R}^m$ is either compact and connected or it is closed and all its connected components are unbounded. On the other hand each set of one of these types is a limit set of some rearranged conditionally convergent series. Moreover, this characterization does not hold in infinite dimensional spaces. We show that if $\sum_{n=1}^{\infty} x_n$ has the Rearrangement Property and A is a closed subset of the closure of the $\sum_{n=1}^{\infty} x_n$ sum range and it is ε-chainable for every $\varepsilon > 0$, then there is a permutation τ such that $A = LIM(\sum_{n=1}^{\infty} x_{\tau(n)})$. As a byproduct of this observation we obtain that series having the Rearrangement Property have closed sum ranges.