TOPOLOGICAL AND ALGEBRAIC PROPERTIES OF SPACES OF LORCH ANALYTIC MAPPINGS

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Abstract. If E is a commutative complex Banach algebra with unit and U is an open (non empty) connected subset of E, a mapping $f : U \subset E \to E$ is Lorch analytic in U if given any $a \in U$ there exists $\rho > 0$ and there exist unique elements $a_n \in E$, such that $B_\rho(a) \subset U$ and $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$, for all z in $\|z-a\| < \rho$. The theory of Lorch analytic mappings goes back to the 1940's (cf. [4]) and is a very natural extension of the classical concept of analytic function to infinite dimensional algebras. Let $f \in \mathcal{H}_L(U,E)$ (the space of all Lorch analytic mappings in an open subset U of E) and $\phi \in \mathcal{M}(E)$ (the set of all complex homomorphisms not identically 0 on E). If there exists a (necessarily unique) complex analytic function $g : \phi(U) \to \mathbb{C}$ so that $g \circ \phi = \phi \circ f$ on U, we say that g is the quotient function of f with respect to ϕ and write $g = f_\phi$. Glickfeld defined the quotient function in [1]. In [3] he discussed the existence of f_ϕ and used this idea to prove two different versions of the inverse function theorem (under different special hypothesis). By using the idea of quotient function, he was also able to obtain a generalization of the Mittag-Leffler's Theorem in [2].

In this talk we will present algebraic and topological results concerning $\mathcal{H}_L(U,E)$ endowed with convenient topologies. Many of these results have been obtained by using the idea of quotient function $f \in \mathcal{H}_L(U,E)$ with respect to $\phi \in \mathcal{M}(E)$.

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