SHORT TALKS

Function Theory on Infinite Dimensional Spaces VIII.

Madrid, 15-18 December 2003.

Index of Abstracts

(In this index, in case of multiple authors only the speaker is shown)

On the Müntz Theorem for numerable compact sets

José María Almira. Universidad de Jaén (Spain)

Abstract. In this note we solve the Müntz problem for the space $\mathbf{C}(K)$ whenever $K \subset [0, \infty)$ is a numerable compact set which satisfies certain additional assumptions and we propose the general case as an open question. By using a well known consequence of the Hahn-Banach theorem, we convert the problem of the density of the span of the monomials $\{x^{\lambda_i}\}_{i=0}^{\infty}$ in a problem about the zero sets of certain analytic functions. The open problem can thus be formulated as a basic question in the theory of complex functions.

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Radon-Nikodým compact spaces and Banach spaces of low weight

Antonio Avilés. Universidad de Murcia (Spain)

Abstract. The concept of Radon-Nikodým compact space has its origin in Banach space theory, and it is defined as a topological space which is homeomorphic to a weak* compact subset of the dual of an Asplund space (that is, a dual Banach space with the Radon-Nikodým property). It is an open problem whether a continuous image of a Radon-Nikodým compact is a Radon-Nikodým compact. We present a new partial answer to this problem: continuous images of

Radon-Nikodým compacta of weight less than cardinal $\mathbf{b}$ are Radon-Nikodým.

Cardinal $\mathbf{b}$ is the least cardinal of a subset ω^ω not contained in a σ-compact subset. It is consistent that $\mathbf{b} > \omega_1$ and the statement $\mathbf{b} = \mathbf{c}$ is weaker than Martin's axiom.

This result on continuous images has a Banach space counterpart: If a Banach space of density character less than $\mathbf{b}$ is a subspace of an Asplund generated space

(respectively a subspace of a WCG space), then it is Asplund generated (respectively WCG). There are counterexamples to this statements of density character exactly **b**.

Foundations of complex Analysis in non locally Convex Spaces

Aboubakr Bayoumi. King Saud University (Saudi Arabia)

Abstract. In fact, real development in Function Theory has been given after we have introduced several new concepts, especially the concept of Quasi-Differentiability which is stronger than Fréchet one. We have built up a complete theory for Function Theory in F-spaces which are not necessarily locally convex.

This lecture will be on a part of our first book entitled: “Foundations of complex Analysis in non locally Convex Spaces” which appear this year.

Operators with common Hypercyclic subspaces

Juan Bes. Bowling Green University (USA)
(Joint with R. Aron, F. Leon and A. Peris)

Abstract. Let T be a bounded operator on a Banach space X. A vector x in X is said to be hypercyclic for T provided the orbit $\{x, Tx, T^2x, \dots\}$ is dense in X. We consider conditions for a family of operators acting on X to have a common hypercyclic subspace, that is, to have a closed subspace of infinite dimension consisting entirely, except for the origin, of vectors that are hypercyclic for each operator in the family. Our work complements recent results by G. Costakis and M. Sambarino and by F. Bayart.

Automatic continuity of directional derivatives

Eusebio Corbacho. Universidad de Vigo (Spain)
Joint work with V. Tarieladze

Abstract. Let f be a mapping defined on an open set G of a Banach space E with values in a Banach space F and $x_0 \in G$.
Definition. The mapping f is said to be *directionally differentiable* $x_0 \in G$ if there is a mapping $T_{x_0,f} : E \to F$ such that

$$T_{x_0,f}(u) = \lim_{t\to 0} \frac{f(x_0 + th) - f(x_0)}{t} \qquad \forall h \in E. \tag{1}$$

If f is directionally differentiable at x_0 and its directional derivative $T_{x_0,f} : E \to F$ is a **continuous linear operator**, then f is said to be *Gâteaux differentiable at* x_0.

We consider two natural questions:

Question 1. Assume that the given **continuous** f is directionally differentiable at x_0 and its directional derivative $T_{x_0,f} : E \to F$ is an **additive mapping**. Is then f **Gâteaux differentiable at** x_0?

Question 2. Is there a wide class Φ of mappings from G to F such that if a given **continuous** $f \in \Phi$ is the directionally differentiable at x_0, then f **Gâteaux differentiable at** x_0?

In the given talk we will discuss the following two results:

Theorem 1. *Question 1 has the affirmative answer.*

Theorem 2. *If G is convex, $F = R$ and Φ consists of* **convex functionals**, *then Question 2 has the affirmative answer as well.*

Theorem 1 seems not to be noticed earlier in the literature. Theorem 2 is known [G, p. 127, Theorem 7].

This talk is based on [BCPT], where more general versions of Theorem 1 and Theorem 2 are contained.

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Approximation of uniformly continuous mappings in Orlicz spaces

Sylvain Delpech. Université de Bordeaux (France)

`Abstract.` We estimate the best exponent α for which every uniformly continuous function, from the unit ball, B_M, of a reflexive Orlicz functions space, endowed with the Luxemburg norm $\|\,.\,\|_M$, into any normed vector space $(Y, \|\,.\,\|_Y)$, can be approximated by α-Hölder functions. We also discuss the sharpness of our estimate. Then we prove that this best α, defined by :

$$\alpha(B_M, Y) = \sup\{\alpha \in [0,1],\ \overline{\mathcal{H}^\alpha(B_M, Y)}^{\|\,.\,\|_\infty} = UC(B_M, Y)\},$$

where $\mathcal{H}^\alpha(B_M, Y)$ is the set of α-Hölder functions between B_M and Y, $UC(B_M, Y)$ is the set of uniformly continuous functions between B_M and Y, may sometimes be only a supremum and not a maximum.

Viscosity solutions to Hamilton-Jacobi equations on Riemannian manifolds

Juan Ferrera. Universidad Complutense de Madrid (Spain)
Joint work with D. Azagra and F. López Mesas

Abstract. We establish some perturbed minimization principles, and we develop a theory of subdifferential calculus, for functions defined on Riemannian manifolds. Then we apply these results to show existence and uniqueness of viscosity solutions to Hamilton-Jacobi equations defined on Riemannian manifolds.

Composition operators on a Hilbert space of Dirichlet Series

Catherine Finet. Université de Mons-Hainaut (Belgium)

Abstract. We study the numerical range and compacity of some composition operators, defined on a Hilbert space of Dirichlet series introduced by Hedenmalm, Linqvist and Seip.
We describe the numerical range of "nice" composition operators. We focus on the zero-inclusion question.
The compacity is studied for symbols

$$\phi(s) = c_0 s + c_1 + \sum_{j=1}^{d} c_{q_j} q_j^{-s} \quad (d = 1, 2, 3)$$

where the q_j's are multiplicatively independent.
The situation is completely different, according whether $c_0 \geq 1$ or $c_0 = 0$.
This is a joint work with H. Queffler and A. Volberg.

Smooth Approximation on Banach Spaces

Robb Fry. Saint Francis Xavier University (Canada)

Abstract. We consider the general problem of uniformly approximating continuous functions on Banach spaces by functions with a higher degree of smoothness.

We shall discuss some of the key classical results, as well as more recent and less known developments.

Polynomials generated by linear operators

Pablo Galindo. Universidad de Valencia (Spain)
Joint work with M. L. Lourenco and L. A. Moraes

Abstract. The class of Banach algebra valued n-homogeneous polynomials generated by the n^{th} powers of linear operators is the topic of this talk. We compare it with the finite type polynomials, as well as their closures in the norm topology.

Optimal Lipschitz Extensions

Thierry Gaspari. Université de Bordeaux (France)

Abstract. Let X be a Banach space, Ω be an open subset of X and $u \in C(\Omega, \mathbb{R})$. We say that u is AML (Absolutely Minimizing Lipschitzian) if for all bounded open set V with $\overline{V} \subset \Omega$, for all $v \in C(\overline{V}, \mathbb{R})$, we have

$$u = v \text{ on } \partial V \Rightarrow \mathrm{Lip}(u, V) \leq Lip(v, V).$$

If u is AML in Ω and equals the function f on the boundary of Ω, then u is called an optimal Lipschitz extension of f in Ω. We study the relationship between the AML functions, the viscosity solutions of the infinity Laplacian partial differential equation, and the functions which satisfy another property called the comparison with cones. We prove that these notions are equivalent in any Hilbert space X. We also study the existence of optimal Lipschitz extensions.

Peripheral spectrum of positive operators and generalizations of Jentzsch's theorem

Arkady Kitover. Community College of Philadelphia (USA)

Abstract. The main result. Theorem. Let X be a Banach lattice and $T : X \to X$ be a positive operator. If the peripheral spectrum of T consists of a finite number of eigenvalues of T then it is cyclic.

Moreover, if X has the Fatou property, T is sequetially order continuous, and for any non-zero positive $u \in X$ Tu is a weak unit in X then the peripheral spectrum consists of only one point $r(T)$ and the corresponding eigenspace is one-dimensional.

Average locally uniformly rotund dual norms

Sebastián Lajara. Universidad de Castilla La Mancha (Spain)

Abstract. Average locally uniformly rotund norms (ALUR for short) are a generalization of LUR norms. They were introduced by Troyanski, who proved that every rotund norm with the Kadec property is ALUR and that every ALUR

space admits an equivalent LUR norm. This result suggests the following problem: *If a dual Banach space is ALUR, does it necessarily admit an equivalent dual LUR norm?* The dual of the James Tree space provides a negative answer to this question (Schachermayer). Therefore, the class of dual LUR spaces is strictly contained in the class of ALUR dual renormable spaces. Banach spaces with equivalent LUR norms were characterized in terms of
countable decompositions of such spaces by Moltó, Orihuela
and Troyanski, and by Raja, who gave a generalization to the dual case. In this note, we provide a covering type characterization of dual Banach spaces with equivalent ALUR dual norms.

Canonical isometries between spaces of polynomials

Silvia Lassalle. Universidad de Buenos Aires (Argentina)

`Abstract.` This talk is concerned with work with Christopher Boyd in which isometries between spaces of homogeneous polynomials are studied. We characterize the case of $\mathcal{P}_A(^nE)$ when E is a real Banach space. In the complex case a better knowledge of the extreme points of the unit ball of $\mathcal{P}_I(^nE)$ is required. The characterization of the isometries of $\mathcal{P}_A(^nE)$ when E is a real Banach space can be extended to give a characterization of the isometries of $\mathcal{P}_A(^nE, X)$ where X is a Banach space with X' strictly convex. This result replies heavily on the idea of centralizer of a Banach space.

Spectra of weighted composition operators on weighted Banach spaces of analytic functions

Joint work with Richard Aron

Mikael Lindström. Åbo Akademi University (Finland)

`Abstract.` We determine the spectra of weighted composition operators acting on the weighted Banach spaces of analytic functions $H^\infty_{v_p}$ when the symbol ϕ has a fixed point in the open unit ball. As an application of this result we give the spectra of composition operators on Bloch type spaces.

Linear polarization constant of $\mathbb{R}^n$

Mate Matolcsi. Renyi Institute for Mathematics (Hungary)

Abstract. In this note we aim to make a contribution to estimating the n-th linear polarization constant $c_n(H)$ of an n-dimensional real Hilbert space H. We recall the

Definition 1 (Benítez, Sarantopoulos, Tonge). *The n-th linear polarization constant of a Banach space X is defined by*

$$\begin{aligned} c_n(X) &:= \inf\{M : \|f_1\| \cdots \|f_n\| \leq M \|f_1 \cdots f_n\| \; (\forall f_1, \dots, f_n \in X^*)\} \\ &= 1 / \inf_{f_1,\dots,f_n \in S_{X^*}} \sup_{\|x\|=1} |f_1(x) \cdots f_n(x)|. \end{aligned}$$

For a real or complex Hilbert space H of dimension at least n, we always have $c_n(H) = c_n(\mathbb{K}^n)$.

Balázs' note: Bení tez, Sarantopoulos and Tonge proved that for isomorphic Banach spaces X and Y we have $c_n(X) \leq d^n(X,Y) c_n(Y)$, where $d(X,Y)$ denotes the Banach-Mazur distance of X and Y. Note, that for any n-dimensional space X a result of John states that $d(X, \mathbb{K}^n) \leq \sqrt{n}$ (where $\mathbb{K}^n$ denotes the n-dimensional Hilbert space). The combination of these results mean that the determination of $c_n(\mathbb{K}^n)$ gives information on the linear polarization constants of other spaces, too.

The exact values of $c_m(\mathbb{K}^n)$ seem very hard to determine. A remarkable result of Arias-de-Reyna states that $c_n(\mathbb{C}^n) = n^{n/2}$. Ball's recent solution of the complex plank problem also implies the same result.

The value of $c_n(\mathbb{R}^n)$ seems even harder to find. The determination of $c_n(\mathbb{R}^n)$, by the definition and the Riesz representation theorem, boils down to determining

$$I := \inf_{x_1,\dots,x_n \in S} \sup_{\|y\|=1} | \langle x_1, y \rangle \cdots \langle x_n, y \rangle |$$

The estimate $I \leq n^{-\frac{n}{2}}$ follows by considering an orthonormal system.

The result of Arias-de-Reyna can be used to derive the following estimates:

$$n^{\frac{n}{2}} \leq c_n(\mathbb{R}^n) \leq 2^{\frac{n}{2}-1} n^{\frac{n}{2}}.$$

A natural, intriguing conjecture is the following.

Conjecture. $c_n(\mathbb{R}^n) = n^{n/2}$.

Marcus gives the following estimate: If $x_1, x_2, \dots, x_n$ are unit vectors in $\mathbb{R}^n$ then there exists a unit vector y such that

$$| \langle x_1, y \rangle \cdots \langle x_n, y \rangle | \geq (\lambda_1/n)^{n/2}, \tag{2}$$

where λ_1 denotes the smallest eigenvalue of the Gram matrix $XX^* = [\langle x_i, x_j \rangle]$. Marcus also expressed the opinion that lower bounds on $\sup_{\|y\|=1} | \langle x_1, y \rangle \cdots \langle x_n, y \rangle |$ should involve the eigenvalues $\lambda_1, \dots, \lambda_n$ of the Gram matrix XX^*, i.e. we should look for estimates of the form $\sup_{\|y\|=1} | \langle x_1, y \rangle \cdots \langle x_n, y \rangle | \geq f(\lambda_1, \dots, \lambda_n) n^{-n/2}$. Note that $\sum_j \lambda_j = \mathrm{Tr}\, XX^* = n$. Therefore the above Conjecture can be equivalently formulated as

$$\sup_{\|y\|=1} | \langle x_1, y \rangle \cdots \langle x_n, y \rangle | \geq 1 \cdot n^{-n/2} = \left(\frac{\lambda_1 + \cdots + \lambda_n}{n} \right)^{n/2} n^{-n/2}.$$

In this note we show that if we replace the arithmetic mean by the harmonic mean of the numbers $\lambda_1, \dots, \lambda_n$, then the corresponding lower estimate does hold.

Theorem. 1. *Assume that the given unit vectors $x_1, \dots, x_n$ are linearly independent. Let $\lambda_1, \dots, \lambda_n$ denote the eigenvalues of the Gram matrix XX^*. Then*

$$\max_{\|y\|=1} \prod_{j=1}^{n} |(Xy)_j| \geq \left(\frac{n}{\lambda_1^{-1} + \cdots + \lambda_n^{-1}} \right)^{n/2} \cdot n^{n/2} \tag{3}$$

This gives an improvement of Marcus' result but the Conjecture still remains open.

Finite-Dimensional Banach Spaces with numerical index zero

Javier Merì. Universidad de Granada (Spain)

Abstract. We prove that a finite-dimensional Banach space X has numerical index 0 if and only if it is the direct sum of a real space X_0 and nonzero complex spaces $X_1, \dots, X_n$ in such a way that the equality

$$\left\| x_0 + e^{iq_1\rho}x_1 + \cdots + e^{iq_n\rho}x_n \right\| = \|x_0 + \cdots + x_n\|$$

holds for suitable positive integers $q_1, \dots, q_n$, and every $\rho \in \mathbb{R}$ and every $x_j \in X_j$ $(j = 0, 1, \dots, n)$. If the dimension of X is two, then the above result gives $X = \mathbb{C}$, whereas $\dim(X) = 3$ implies that X is an absolute sum of $\mathbb{R}$ and $\mathbb{C}$. We also give an example showing that, in general, the number of complex spaces cannot be reduced to one.

El operador de Volterra no es superciclico.

Alfonso Montes. Universidad de Sevilla (Spain)

Abstract. En esta charla veremos un esbozo de la prueba de que el operador de clasico de Volterra no es superciclico. En la prueba se hace un uso especial de los polinomios de Legendre. Trabajo conjunto con Eva Gallardo.

Deleting diffeomorphisms and starlike bodies in Banach spaces

Alejandro Montesinos. Universidad Complutense de Madrid (Spain)

Abstract. This talk aims to show the theory of topological negligibility in its historical context, and to explore, with some detail, both the way in which it has developed and its interesting applications in several branches of mathematics. Given a topological space X and a subset K of X, we say that K is topologically

negligible if there exists a homeomorphism between X and $X \setminus K$. This homeomorphism, which we call *deleting*, is usually required to be the identity outside a given neighborhood of K. In 1953, V. Klee proved that if X is a non-reflexive Banach space or an infinite-dimensional L^p space, every K compact subset of X is topologically negligible. Later on, several mathematicians (C. Bessaga, T. Dobrowolski and others) studied the smooth negligibility, where, by discarding deleting homeomorphims, the attention is paid to the notion of deleting diffeomorphims. For the time being, the deepest results about smooth negligiblity are due to D. Azagra and T. Doborwolski, who showed that for every Banach space $(X, \|\cdot\|)$ with a C^p-smooth norm ρ, compacta of X are negligible via a C^p-diffeomorphism. Now, in a joint work with D. Azagra, we have proved, by exploiting starlike bodies, some other results that contribute to understand in what Banach spaces, compacta and points are smooth negligible. For instance, we have proved that in every Banach space X with smooth partitions of unity, weak compacta of X are smooth negligible. In addition, it has been established that if X is an infinite-dimensional separable Banach space with a Schauder basis, and X has a C^p- smooth bump function, then for every compact subset K and every open subset U of X with $K \subset U$, there exists a C^p-diffeomorphism $h : X \to X \setminus K$ such that h is the identity on $X \setminus U$.

Finally, it is important to remark that this kind of results about topological and smooth negligibility has found interesting applications in several branches of mathematics, which include fixed point theory, strange phenomena concerning EDO's, and approximate strong versions of the Morse-Sard theorem.

Compacta in product of lines from a functional-analytic point of view

Vicente Montesinos. Universidad de Valencia (Spain)
Joint work with M. Fabian, G. Godefroy and V. Zizler

`Abstract.` Along several papers we developed a technique of splitting certain subsets of a Banach space X in such a way that it allows us to detect whether or not X belongs to one of several classes of non-separable Banach spaces. Those subsets sometimes are Markushevich bases, sometime just total and countably supported subsets. Classes of Banach spaces considered included the weakly compactly generated, the subspaces of weakly compactly generated, the Vašák or weakly countably determined and the weakly Lindelöf determined. The technique is reminiscent of the way several authors (S. Argyros, S. Mercourakis, V. Farmaki, S. P. Gu'lko, G. A. Sokolov,...) used to split the set of coordinates in representing compact sets as subsets of products of lines. It is not surprising, then, that we can recover those representations by using our approach. This is done here for the Eberlein and the Gul'ko compacta.

Vector valued Sobczyk's theorem

Yolanda Moreno. Universidad de Extremadura (Spain)

Abstract. We provide a vectorial version of Sobczyk's theorem. Recall that Sobczyk's theorem can be read as follows: every w^*-null sequence of functionals $(y_n^*)_n$ on a Banach space, say Y, can be extended to any space X such that X/Y is separable by a w^*-null sequence of functionals. We prove a vectorial version of such result, essentially: every uniformly bounded sequence of operators $T_n : Y \to Y_n$ admitting uniformly bounded extensions to a space X such that X/Y is separable and in such a way that (T_n) converges to 0 in the strong operator topology (SOT) can be extended to X by a SOT-null sequence. In particular, Sobczyk's theorem and the Rosenthal and Johnson-Oikhberg's results related to the topic are obtained as consequence.

Kuratowski's index of non-compactness and renorming in Banach spaces

Luis Oncina. Universidad de Murcia (Spain)
Joint work with F. García, J. Orihuela and S. Troyanski

Abstract. A point $x \in A \subset (X, \|\cdot\|)$ is *quasi-denting* if for every $\varepsilon > 0$ there exists a slice of A containing x with Kuratowski index less than ε, where the *Kuratowski index of non-compactness of B* is defined by $\alpha(B) := \inf\{\varepsilon > 0 : B$ is covered by a finite family of sets of diameter less than $\varepsilon\}$.

Theorem. *Let X be a normed space and let F be a norming subspace of its dual. Then X admits an equivalent $\sigma(X, F)$-lower semi-continuous* **LUR** *norm if, and only if, for every $\varepsilon > 0$ we can write $X = \cup_{n=1}^{\infty} X_{n,\varepsilon}$ in such a way that for every $x \in X_{n,\varepsilon}$, there exists a $\sigma(X, F)$-open half space H containing x with $\alpha(H \cap X_{n,\varepsilon}) < \varepsilon$.*

Taking advantage of homogeneity one can replace the space X in the former theorem by its unit sphere.

¿From the topological point of view we shall see that a normed space X admits an equivalent $\sigma(X, F)$-lower semi-continuous **LUR** norm if, and only if, the norm topology has a network $\mathcal{N} = \cup_{n=1}^{\infty} \mathcal{N}_n$ such that for every $n \in \mathbb{N}$ and for every $x \in \cup \mathcal{N}_n$ there is a $\sigma(X, F)$-open half space H with $x \in H$ such that H meets only a finite number of elements from $\mathcal{N}_n$, therefore turning the 'discrete' condition appearing in [A. Moltó, J. Orihuela, S. Troyanski, and M. Valdivia, *On weakly locally uniformly rotund Banach spaces*, J. Funct. Anal. **163** (1999), no. 2, 252–271.] into a 'locally finite' one.

Spectra of composition operators on BMOA

Niklas Palmberg. Åbo Akademi University (Finland)
Joint work with Mikael Lindström (Åbo Akademi University)

Abstract. It is shown that if ϕ is an univalent self map on the unit disk D, not an automorphism, with a fixed point in D and the essential spectral radius of the composition operator C_ϕ on H^2 is different from zero, then the spectrum of C_ϕ on BMOA coincides with $\overline{D}$. This answers in the affirmative a conjecture by MacCluer and Saxe.

Absolutely Summing Multilinear Operators

David Pérez. Universidad Rey Juan Carlos (Spain)

Abstract. We give a new definition of absolutely summing multilinear operator and we see that it seems to be the right one, in the sense that we can generalize most of the linear results. In particular, we have multilinear versions of Grothendieck's Theorem and good relations with some other well known classes of multilinear operators, such as Hilbert-Schmidt, nuclear or integral operators.

Hamiltonian structure of the harmonic functions on an annulus

Luis M. Pozo Coronado. Universidad Complutense de Madrid (Spain)
Joint work with J. Muñoz Masqué

Abstract. We consider the space of harmonic functions on an n-dimensional annulus, seen as the space of extremals of the energy functional. The variational approach allows us to find an alternate bilinear form which is proved to be nondegenerate, thus endowing the harmonic functions with an structure of a symplectic manifold.

On lineability of sets of unimaximal continuous functions

Lucas Quarta. Université de Mons-Hainaut (Belgium)

Abstract. A set M in a linear topological space X is said to be *n-lineable* (resp. *lineable*, resp. *spaceable*) in X if $M \cup \{0\}$ contains a vector space of dimension n (resp. infinite dimensional, resp. closed and infinite dimensional). If the maximum cardinality of such a vector space exists it is called the *lineability* of M. One can find in [1] a lot of results of "linearity in non linear problems" in many different fields of analysis. One of the first works in this spirit is the lineability of the set of nowhere differentiable functions on $[0,1]$,[6] (see also [4], [7] and [8]). Recently, several papers were devoted to the study of the lineability of sets of functions on $[0,1]$ or $\mathbb{R}$ which satisfy other special properties (see for example [2] and [3]). Our goal is the study of the lineability of the set $M = \hat{C}(S)$ of continuous functions

defined on a set S and which admit one and only one absolute maximum. We will present an analytical approach when S is a subset of $\mathbb{R}$ (joint work with V.I. Gurariy) and a geometrical one (based on ideas introduced in [5]) in higher dimensions (joint work with C. Troestler).

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Polarization constants, plank problems, and local theory of Banach spaces

Szilard Revesz. Renyi Institute for Mathematics (Hungary)

Joint work with Y. Sarantopoulos

Abstract.

Definition 2 (Benítez - Sarantopulos - Tonge). *The n^{th} (linear) polarization constant of a Banach space X is defined by*

$$c_n(X) := \inf\{M > 0 : \|f_1\| \cdots \|f_n\| \le M \cdot \|f_1 \cdots f_n\|, \forall f_1, \dots, f_n \in X^*\}\,. \tag{4}$$

Moreover, the (linear) polarization constant of X is $c(X) := \limsup_{n\to\infty} c_n(X)^{1/n}$.

If $f_1, f_2, \dots, f_n \in X^*$, then the product $(f_1 f_2 \cdots f_n)(x) := f_1(x) f_2(x) \cdots f_n(x)$ is a continuous n-homogeneous polynomial on X. In 1998 R.A. Ryan and B. Turett proved the existence of a universal bound C_n of (4), and Benítez, Sarantopoulos and Tonge proved that in the case of *complex* Banach spaces, $C_n \le n^n$ and the constant n^n is best possible. Clearly, $c_n(\cdot)$ and $c(\cdot)$ are isometric properties of Banach spaces, also fitting into the local theory of Banach spaces, since for any X we have $c_n(X) = \sup\{c_n(Y) : Y \subseteq X, \dim Y = n\}$.

Proposition 1. *For $c(X)$ also the limit exists and we have*

$$c(X) = \lim_{n\to\infty} c_n(X)^{1/n}\,.$$

The growth of $c_n(X)$ when $n \to \infty$ is characteristic to the space X. Eg. we have

Theorem. 2. *Let X be any normed space. Then $c(X) = \infty$ iff $\dim(X) = \infty$.*

Although it seems to be very likely, but it is not known whether $c_n(X) \geq c_n(\ell_2^n)$ $(\forall n \leq \dim X)$ holds true in general. Still, for each $n \in \mathbb{N}$, Hilbert spaces have the smallest n^{th} polarization constant among *infinite dimensional* normed spaces.

Theorem. 3. *If $\dim X = \infty$, then $c_n(X) \geq c_n(\ell_2^n)$, $\forall n \in \mathbb{N}$.*

In this work we discuss various results, estimates and exact values found for linear polarization constants of Banach, and in particular of Hilbert spaces. We also present expected and unexpected connections to the plank problem of Tarski, to Chebyshev constants, and to various well-known polynomial inequalities. In this respect, one of the challanging problems of the subject is the following conjecture, which seems to be widely believed.

Conjecture [Sarantopoulos]. $c_n(\mathbb{R}^n) = n^{\frac{n}{2}}$.

Note that Arias-de Reyna proved $c_n(\mathbb{C}^n) = n^{\frac{n}{2}}$, and an even more precise result was found by K. Ball recently. We will explain, why the real case is more difficult.

The Birkhoff integral

José Rodríguez. Universitad de Murcia (Spain)

`Abstract.` There are several extensions of Lebesgue's theory of integration to the case of functions with values in Banach spaces, being widely known those due to Bochner and Pettis. Birkhoff integral lies strictly between the Bochner and Pettis integrals and has caught the attention of some authors pretty recently. This talk will be devoted to show the new results about this topic that appear in our papers: "Birkhoff integral and the property of Bourgain" (with B. Cascales) and "On the existence of Pettis integrable functions which are not Birkhoff integrable".

We prove that there is a close relationship between Birkhoff integrability and Bourgain property: a bounded function f defined on a complete probability space with values in a real Banach space X is Birkhoff integrable if, and only if, the set of compositions of f with elements of the dual unit ball has Bourgain property. This fact allows us to characterize the weak Radon-Nikodým property in dual Banach spaces via Birkhoff integrable Radon-Nikodým derivatives. As a consequence we give a solution, in the case of dual Banach spaces, to a problem posed by Fremlin concerning the representation of vector measures as indefinite Pettis integrals of generalized McShane integrable functions.

It is well known that when the range Banach space is separable, then Birkhoff and Pettis integrability coincide. We try to go a bit further when studying the differences between Birkhoff and Pettis integrability in non-separable Banach spaces. We prove that for a weakly Lindelöf determined Banach space X with density character greater than or equal to the continuum, there exists a bounded Pettis integrable function $f : [0, 1] \longrightarrow X$ which is not Birkhoff integrable. A particular case

of our constructions shows that the analogue of Lebesgue's dominated convergence theorem for the Birkhoff integral does not hold in general.

Strictly singular inclusions between symmetric sequence spaces

Víctor M. Sánchez. Universidad Complutense de Madrid (Spain)
Joint work with F.L. Hernández and E.M. Semenov

Abstract. Strict singularity of inclusions between symmetric sequence spaces will be studied. Suitable conditions will be provided involving the associated fundamental functions. The special case of Lorentz and Marcinkiewicz spaces will be characterized. It will be also showed that if $E \hookrightarrow F$ are symmetric sequence spaces with $E \neq \ell_1$ and $F \neq c_0$ and ℓ_∞ then there exist a intermediate symmetric sequence space G such that $E \hookrightarrow G \hookrightarrow F$ and both inclusions are not strictly singular. As a consequence new characterizations of the spaces c_0 and ℓ_1 inside the class of all symmetric sequence spaces will be given.

Banach Spaces and Polytopes

Antonio Santos. Universidad de Vigo (Spain)

Abstract. Let X be a Banach Space and $B(X)$ the unit ball of X. If $B(X)$ is a polyhedron then $B(X^*)$ is a polyhedron? Where $B(X^*)$ is a unit ball of the dual space.

Construction of Infinite Dimensional Vector Spaces of Hypercyclic Vectors

Juan B. Seoane. Kent State university (USA)
Joint work with Richard M. Aron (Kent State University)

Abstract. Consider $X = \ell_p$ or c_0, and define the weighted backward shift B_λ ($|\lambda| > 1$) on X,

$$B_\lambda(x_1, x_2, x_3, \dots) = \lambda \cdot (x_2, x_3, x_4, \dots).$$

We give a different proof that the set of hypercyclic vectors for this operator is lineable, i.e. there exists an infinite dimensional hypercyclic subspace for B_λ. We also give a technique to construct infinite dimensional common hypercyclic subspaces for a countable family of these operators.
This is a well known result, but we won't use the *Baire category theorem*, we prove it in an explicit and constructive way.

Strong minimality of Gaussian integrals

Ricardo Vidal Vázquez. Universidad de Vigo (Spain)
Joint work with V. Tarieladze

Abstract. Let E be a be a finite-dimensional Hilbert space with $\dim(E) \geq 1$, $\mathcal{U}(E)$ be the set of all isometric linear operators $u : E \to E$ and γ be the standard Gaussian measure on E.

We will discuss some applications of the next statement which expresses "the strong minimality" of the Gaussian integrals.

Proposition. Let μ be a positive measure (not necessarily finite) given on the Borel σ-algebra of E such that

$$\int_E \|x\|^2 d\mu(x) = 1.$$

Then for any positively 2-homogeneous Borel measurable function $f : E \to C$ with $f(0) = 0$, we have:

$$\int_E |f(x)| d\gamma(x) \leq dim(E) \sup_{u \in \mathcal{U}(E)} \int_E |f(ux)| d\mu(x)$$

holds.
This talk is based on [TV].

REFERENCES

[TV] Tarieladze V., Vidal, R. *Strong minimality of Gaussian summing norm*, Acta Mathematica Vietnamica, Vol. 28, Number 2, 2003, pp. 225-240.

Where do homogeneous polynomials on ℓ_1^n attain their norm?

Ignacio Villanueva. Universidad Complutense de Madrid (Spain)
Joint work with David Pérez (Universidad Rey Juan Carlos)

Abstract. In the "Open Problems" session of the conference 'Function Theory on Infinite Dimensional Spaces VII', held in Madrid in 2001, Professor I. Zalduendo asked the question of 'how many' homogeneous polynomials will attain their norm at the vertices of the unit ball of ℓ_∞^n when n tends to infinity. He conjectured that 'almost everyone'. In this direction, he and D. Carando published recently a paper giving qualitative general results. As they say in the introduction, the question is to study how likely it is for a polynomial $P : E \longrightarrow \mathbb{R}$ to attain its norm at a given subset A of the unit ball B_E.

In our talk we present quantitative results referring to 2-homogeneous poynomials on ℓ_1^n, as an example of the results that can be expected in more general cases: Using a 'reasonable' measure in $\mathcal{P}(^2\ell_1^n)$, the space of 2-homogeneous polynomials on ℓ_1^n, we show the existence of a set of positive (and independent of n) measure of

polynomials which do not attain their norm at the vertices of the unit ball of ℓ_1^n. Next we prove that, when n grows, almost every polynomial attains its norm in a face of 'low' dimension.