ALGEBRAIC STRUCTURES FOR FAMILIES OF HYPERCYCLIC ENTIRE FUNCTIONS

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An operator $T \in L(\mathcal{H}(\mathbb{C}))$ is said to be a *convolution operator* provided that it commutes with translations, that is,

$$T \circ \tau_a = \tau_a \circ T \quad \text{for all} \quad a \in \mathbb{C},$$

where $\tau_a f := f(\cdot + a)$. Then $T \in L(\mathcal{H}(\mathbb{C}))$ happens to be a convolution operator if and only if T is an infinite order linear differential operator with constant coefficients $T = \Phi(D)$, where Φ is an entire function with *exponential type*, that is, there exist constants $A, B \in (0, +\infty)$ such that $|\Phi(z)| \leq Ae^{B|z|}$ for all $z \in \mathbb{C}$.

For an entire function of exponential type $\Phi(z) = \sum_{n \geq 0} a_n z^n$ and $f \in \mathcal{H}(\mathbb{C})$, we have

$$\Phi(D)f = \sum_{n=0}^{\infty} a_n f^{(n)}.$$

Godefroy and Shapiro [7] proved that every non-scalar convolution operator is hypercyclic, so covering the classical Birkhoff and MacLane results on hypercyclicity of the translation operator τ_a (take $\Phi(z) = e^{az}$, $a \neq 0$) and of the derivative operator $D : f \mapsto f'$ (take $\Phi(z) = z$), respectively.

Inspired in [1], Bayart and Matheron proved that there is even a residual set of functions in $\mathcal{H}(\mathbb{C})$ generating a hypercyclic algebra for the derivative operator, that is every non-null function in one of these algebras is hypercyclic for the operator D [2, Th. 8.26]. A similar result, with a different approach, was independently obtained by Shkarin [8].

This lead to the following question raised by Aron: *For which functions Φ of exponential type, does $\Phi(D)$ support a hypercyclic algebra?* We provide a partial answer to the above mentioned question by showing the existence of hypercyclic algebras for several convolution operators induced either by polynomials or by transcendental functions [3, 4, 5].

In this line, we also prove the existence of an infinitely generated multiplicative group consisting of entire functions that are, except for the constant function 1, hypercyclic with respect to the convolution operator associated to a given entire function of exponential type. A certain stability under multiplication is also shown [6].

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