TWO EXTENSIONS OF SELF-EXPANDED MAPS

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Our aim is to present a joint work with A. Daniilidis and E. Durand-Cartagena on two generalizations of the notion of self-expanded maps and to study the properties of such maps. Let us recall that if (M, d) be a metric space and I be an interval, a curve $\gamma : I \to M$ is self-expanded if, for all $\tau \in I$, the map $t \mapsto d(\gamma(t), \gamma(\tau))$ is non decreasing on $I \cap [\tau, +\infty)$. If the curve $\gamma : I \to \mathbb{R}^n$ has right derivative - denoted γ' - at each point, then γ is self-expanded if and only if, for all t, τ in I such that $t < \tau$, we have $\langle \gamma'(\tau), \gamma(t) - \gamma(\tau) \rangle \leq 0$. Whenever M is a compact subset of a Riemannian manifold, or whenever M is a compact subset of a finite dimensional vector normed space, all self-expanded maps with values in M have finite length. We introduce two definitions generalizing the notion of self-expanded maps. Given $\lambda \in [0, 1[$ and an interval I, a curve $\gamma : I \to \mathbb{R}^d$ is called a λ-curve if for every $t_1 \leq t_2 \leq t_3$ in I we have

$$d(\gamma(t_1), \gamma(t_2)) \leq d(\gamma(t_1), \gamma(t_3)) + \lambda d(\gamma(t_2), \gamma(t_3))$$

Bounded λ-curves in the euclidean space have finite length whenever $\lambda < 1/d$.

A continuous curve γ, having right derivative at each point, is a λ-eel if, for every $(t, \tau) \in I^2$ such that $t < \tau$,

$$\langle \gamma'(\tau), \gamma(t) - \gamma(\tau) \rangle \leq \lambda \; ||\gamma'(\tau)|| \; ||\gamma(t) - \gamma(\tau)||$$

If $\alpha = \arccos(\lambda)$, $\gamma : I \to \mathbb{R}^n$ is a λ-eel if, for every $\tau \in I$, the open cone $C(\tau, \alpha)$ of origin $\gamma(\tau)$, direction $\gamma'(\tau)$ and of angle α does not meet $\Gamma(\tau)$. A λ-curve is a λ-eel, but the converse is false. Whenever $\lambda \geq \frac{1}{\sqrt{5}}$, we present the construction of a λ-eel γ with values in the unit ball of $\mathbb{R}^3$, and of infinite length. On the other hand, if $\gamma : I \to \mathbb{R}^2$ is a λ-ell for some $\lambda < 1$, and if $\gamma(I)$ is bounded, then the length of γ is finite.