# EXTENDING BILINEAR MAPS ON BANACH SPACES BY HOMOLOGY

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Abstract. Given two Banach spaces $X$and $Y$ let $\mathcal{L}(X,Y)$ denote the vector space of linear continuous operators acting between them; its derived functor is the one that assigns to each couple $X;Y$ the vector space $\mathrm{Ext}(X,Y)$ of exact sequences $0 \to Y \to \square \to X \to 0$ modulo equivalence; let us agree that the second derived functors will be called $\mathrm{Ext}^2(X,Y)$.

Several important Banach space problems and results adopt the form $\mathrm{Ext}(X,Y) = 0$ (or $\mathrm{Ext}(X,Y) = 0$). For instance,

- Sobczyk's theorem: $\mathrm{Ext}(c_0, X) = 0$ for every separable Banach space $X$.
- Lindenstrauss's lifting principle: $\mathrm{Ext}(L_1(\mu), X) = 0$ for every ultrasummand $X$.
- The Enflo-Lindenstrauss-Pisier and Kalton-Peck construction: $\mathrm{Ext}(\ell_2, \ell_2) \neq 0$.
- The Johnson-Zippin's theorem: $\mathrm{Ext}(H^*, \mathcal{L}_\infty) = 0$ for every subspace $H$ of $c_0$.

In general, a basic Banach space question is whether $\mathrm{Ext}(X,Y) = 0$ for a given couple of Banach spaces $X;Y$. Similar questions for $\mathrm{Ext}^2$ have not been treated. Let us write $\mathrm{Ext}^2(X,Y) = 0$ to mean that all elements $FG$ of $\mathrm{Ext}^2(X,Y)$ are 0.

Palamodov's Problem 6 in [2] says: Is $\mathrm{Ext}^2(\cdot, E) = 0$ for any Fréchet space? Let us answer it in the negative even in the domain of Banach spaces. Perhaps the most interesting situation is the Hilbert space case:

**Problem.** Is $\mathrm{Ext}^2(\ell_2, \ell_2) = 0$ ?

for which a few partial results can be obtained. The following unexpected connection wich extension of bilinear forms is proved in [1]:

**Theorem.** $\mathrm{Ext}^2(\ell_2, \ell_2) = 0$ if and only if whenever $\ell_1/D_2 = \ell_2$, every bilinear form defined on $D_2$ can be extended to a bilinear form on $\ell_1$.

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## References

[1] J.M.F. Castillo and R García, Bilinear forms in the homology of Banach spaces, preprint 2017.
[2] V. Palamodov, The projective limit functor in the category of topological linear spaces. (Russian) Mat. Sb. (N.S.) 75 (117) 1968, 567–603 (English Transl. Math-USSR-Sb 4 (1968) 529-558).