ON A PROBLEM BY GURARIY CONCERNING SUBSPACES OF CONTINUOUS FUNCTIONS

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Abstract. Let $A \subseteq \mathbb{R}$ and denote by $\widehat{\mathcal{C}}(A)$ the subset of $\mathcal{C}(A)$ of functions attaining their maximum at a unique point. In 2004 V. I. Gurariy and L. Quarta proved that the set $\widehat{\mathcal{C}}([0,1]) \cup \{0\}$ does not contain a 2-dimensional space whereas $\widehat{\mathcal{C}}([0,1)) \cup \{0\}$ and $\widehat{\mathcal{C}}(\mathbb{R}) \cup \{0\}$ do contain a 2-dimensional space. Using the usual teminology in lineability theory, we can say that $\widehat{\mathcal{C}}([0,1])$ is not 2-lineable whereas $\widehat{\mathcal{C}}(\mathbb{R})$ and $\widehat{\mathcal{C}}([0,1))$ are 2-lineable. During a Non-linear Analysis Seminar held at Kent State University in the academic year 2003/2004, V. I. Gurariy posed the following question: Is $\widehat{\mathcal{C}}(\mathbb{R})$ (or equivalently $\widehat{\mathcal{C}}([0,1))$) n-lineable for $n \geq 3$? The answer to this question has resisted the efforts of many mathematicians ever since. Using a topological approach based on Moore's Theorem we have been able to prove that $\widehat{\mathcal{C}}(\mathbb{R})$ is not three lineable.
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