THE SETS OF MONOMIAL CONVERGENCE FOR POLYNOMIALS AND HOLOMORPHIC FUNCTIONS

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Abstract. There are two possible ways to approach holomorphy: the Cauchy way through differentiability and the Weierstrass way through analyticity. These approaches coincide for functions of several complex variables but in infinite many dimensions they are different.

We are interested in functions on the open unit ball B_{c_0} of the Banach space c_0 of the null sequences. Here for each holomorphic function f we have a unique family of coefficients $(c_\alpha(f))_{\alpha\in\mathbb{N}_0^{(\mathbb{N})}}$ such that the formal monomial series $\sum_{\alpha\in\mathbb{N}_0^{(\mathbb{N})}} c_\alpha(f)z^\alpha$ equals $f(z)$ for all finite sequences $z \in B_{c_0}$.

Thus it is natural to define the set of monomial convergence of the space $H_\infty(B_{c_0})$ of all bounded holomorphic functions on B_{c_0}

$$\mathrm{mon}H_\infty(B_{c_0}) := \{z \in \mathbb{C}^{\mathbb{N}} : \sum_{\alpha\in\mathbb{N}_0^{(\mathbb{N})}} |c_\alpha(f)z^\alpha| < \infty \ \text{ for all } \ f \in H_\infty(B_{c_0})\},$$

and similarly for the space $\mathcal{P}_m(c_0)$ of all m-homogeneous continuous polynomials on c_0.

In this talk we are going to study these sets giving a complete description in the case of the m-homogeneous polynomials. We also extend this study to more general situations. We describe the sets of monomial convergence in Banach sequence spaces X for the cases of bounded holomorphic functions on the open unit ball of X and m-homogeneous polynomials on X, being of particular interest the spaces ℓ_p, for $1 \leq p < \infty$. But we go further giving a rather accurate lower bound for $\mathrm{mon}H(R)$, where R is an arbitrary Reinhardt domain in an arbitrary Banach sequence space. This cycle of ideas is related with Bohr's original absolute convergence problem, as well as Bohr radius and unconditionality for spaces of homogeneous polynomials.

The content of this talk is part of the book *Dirichlet Series and Holomorphic Functions in High Dimensions,* New Mathematical Monographs 37. Cambrigde University Press (2019) by A. Defant, M. Maestre, P. Sevilla-Peris and myself.