# GLOBAL GEOMETRY AND $C^1$ CONVEX EXTENSIONS OF JETS

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Abstract. Given $E \subset \mathbb{R}^n$ arbitrary, and two functions $f : E \to \mathbb{R}$, $G : E \to \mathbb{R}^n$, it is natural to look for necessary and sufficient conditions on $f, G$ for the existence of a $C^1(\mathbb{R}^n)$ convex extension $F$ of $f$ with $\nabla F = G$ on $E$. When $E$ is bounded, the solution to this problem is given by two simple (and natural) conditions and the extension can be taken to be Lipschitz and coercive. However, if $E$ is unbounded these two conditions are no longer sufficient and the problem is much more complicated due to some geometrical aspects of differentiable convex functions (jets) and, in particular, to the presence of *corners at infinity*. The purpose of this talk is to understand the global geometry of convex functions (jets) of class $C^1$ and to show the solution to the extension problem in full generality. Also, we will see that if $G$ is bounded, $F$ can be taken so that $\mathrm{Lip}(F) \lesssim \|G\|_\infty$. As an application, we solve a similar problem about finding $C^1$ convex hypersurfaces with prescribed normals at the points of an arbitrary subset of $\mathbb{R}^n$. This is a joint work with Daniel Azagra.