ON COARSE-LIPSCHITZ EMBEDDINGS OF c_0 INTO DUAL BANACH SPACES

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Abstract. The space c_0 is known to embed badly into separable dual spaces. This is certainly true for linear embeddings, where Bessaga and Pełczyński teach us that $c_0 \subset X^*$ implies that ℓ_1 is complemented in X (and so X^* is non-separable). This is also true for Lipschitz embeddings since Heinrich and Mankiewicz can transform them very well into linear ones by differentiation. Relaxing further the conditions we impose on our embedding, we enter the domain of open problems but, thanks to Kalton, the following is known: if c_0 embeds coarsely into X, then some iterated dual $X^{(n)}$ is non-separable. It is not known whether n could be taken equal to 2 in this result. Tightening a bit the conditions we impose on the embedding, we can show the following.

Theorem 1. *If c_0 coarse-Lipschitz embeds into X^* then the Szlenk index of X is at least ω^2.*

This is still a far cry from the desired result that in this case X^* should be non-separable (which would be, in terms of the Szlenk index, $Sz(X) = \omega_1$). So we tighten a bit again:

Theorem 2. *If c_0 embeds coarse-Lipschitz into X^* with distortion strictly less than $\frac{3}{2}$, then $\ell_1 \subset X$ (and so X^* is non-separable).*

In order to prove Theorem **??** we use the so called Kalton's graphs. A striking application of the pre-duality theory of Lipschitz-free spaces by García Lirola, Petitjean, Rueda Zoca and the author shows that the method of Kalton's graphs cannot go beyond $Sz(X) = \omega^2$. Conversely, we will also discuss some applications of (the proof) of Theorem **??** to the pre-duality theory of Lipschitz free spaces.

These results are coming from a joint work with B. Braga, G. Lancien and C. Petitjean.