

RENORMING WITH GENERALIZED TYPES AND COTYPES

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ABSTRACT. The classic notions of type and cotype have been proved of great utility in many chapters of Banach space theory, including non-linear theory. Indeed, these notions can be formulated in purely metric terms and thus extended to metric spaces (Enflo, Mendel-Naor). However, the classic type and cotype live in a linear scale whereas the complexity of Banach spaces cannot.

We retrieve the quite unnoticed notions of generalized type and cotype in relation with the possibility of improving, by renorming, beyond the linear scale the moduli of convexity and smoothness of super-reflexive spaces. Building on the pioneering work of Figiel we prove that a UMD space can be renormed to have a modulus of convexity not worse than a given generalized cotype.

The limitations for the existence of an optimal modulus of smoothness or convexity are essentially the same that for the existence of best type or cotype. We discuss the properties of the boundary functions and its relation to ordinal indices. This is part of a joint work with Luis C. Garca-Lirola.