RECENT RESULTS WITHIN NASH-MOSER-EKELAND THEORY

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Abstract. Nash-Moser Theorem is an inverse function theorem in Fréchet spaces, see [5]. It is used in problems with infinitely smooth data and loss of derivatives. Its proof uses Newton type method.

In its seminal work [1] Ekeland shows that certain surjectivity can be proved for functions which are only Gâteaux differentiable.

The results in this talk are set out in three common works with Milen Ivanov that further Nash-Moser-Ekeland Theory.

In [2] we prove surjectivity result for a multivalued map. However, when translated to a function, it requires *strong* directional derivatives.

The spaces, which are interesting for applications, e.g. spaces of infinitely differentiable functions, have Heine-Borel property. This can be used to prove in a straightforward way surjectivity using the standard definition of differentiability. Namely, we have the following result.

Theorem 1. *Let $X = \cap_0^\infty (X_n, \|\cdot\|_n)$ and $Y = \cap_0^\infty (Y_n, |\cdot|_n)$ be Fréchet spaces with Heine-Borel property. Let $f : X \to Y$ be continuous, Gâteaux differentiable and such that $f(0) = 0$.*

Assume that there are $c_n > 0$ and $d \in \{0\} \cup \mathbb{N}$, such that for each $x \in X$ and $v \in Y$

$$\exists u \in X : \ f'(x)u = v \text{ and } \|u\|_n \leq c_n |v|_{n+d}, \quad \forall n \geq 0.$$

Then for each $y \in Y$ there is $x \in X$ such that

$$f(x) = y \text{ and } \|x\|_n \leq c_n |y|_{n+d}, \quad \forall n \geq 0.$$

In [4] we combine some ideas from [2] and [3] to generalize the main result of [5].

References

[1] I. Ekeland, An inverse function theorem in Fréchet spaces, Ann. Inst. H. Poincaré C, 28, 2011, Issue 1, 91–105.

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[3] M. Ivanov and N. Zlateva, A Simple Case within Nash-Moser-Ekeland Theory, Compt. rend. Acad. bulg. Sci., 72, 2019, No 2, 152–157.

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[5] R. S. Hamilton, The Inverse Theorem of Nash and Moser, Bulletin of AMS, 7, 1982, Issue 1, 65–222.