EMBEDDINGS OF KALTON'S INTERLACED GRAPHS INTO DUAL BANACH SPACES

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ABSTRACT. The work we will present concerns the non-linear geometry of Banach spaces. One goal in this theory is to classify Banach spaces with the help of some non linear maps (Lipschitz, coarse-Lipschitz, etc). In other words, given two Banach spaces X and Y, one tries to determine whether they are equivalent with respect to a certain category. Sometimes we try to determine if X is equivalent to a subset of Y; when it is true we say that X embeds into Y. A natural and powerful approach to classify mathematical objects is to discover properties that are invariant with respect to isomorphisms or embeddings.

In a fundamental paper on the coarse geometry of Banach spaces (published in 2007), N. Kalton introduced a property that he named property Q. In particular, it is a coarse-invariant and serves as an obstruction to coarse embeddability into reflexive spaces. This property is related to the behavior of Lipschitz maps defined on a particular family of metric graphs: the Kalton interlaced graphs.

In this talk we pursue the study of property \mathcal{Q} and of the interlaced graphs in the particular case of dual Banach spaces X^* . Specifically, we establish some links with the Szlenk index of X and give concrete examples. We also study a weaker version of property \mathcal{Q} which as an application permits us to rule out the classification of some Banach spaces (e.g. ℓ_p spaces and the James spaces \mathcal{J}_p).