DESCRIBING MULTIPLICATIVE CONVEX FUNCTIONS

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ABSTRACT. The theory of convex functions keeps playing a central role in operator theory, in real analysis and in some realms of applied mathematics, such as management science or optimization theory. Since the beginning of their study by Jensen, they have been thoroughly described and many of their properties have been unveiled.

The study of convex functions has extended to the consideration of other inequalities. In this direction, Niculescu proposed the definition of **multiplicative convex functions** (see [1]), since they are supposed to substitute the arithmetic mean in the inequality that defines the convex functions by the geometric mean. Yet, the definition of the multiplicative convex functions could be regarded as a way of upgrading the operations that take part in the definition of convex functions. In this direction, we propose the following variation for the definition of multiplicative convex functions:

Definition 0.1. *Let $f : (0, \infty) \to [0, \infty)$ be such that $f(1) = 1$. We will say that f is* **multiplicative convex** *if, for every $\mu > 0$ and $x, y \geq 0$ we have*

$$f(x^\mu y^{1/\mu}) \leq f(x)^\mu f(y)^{1/\mu}. \tag{1}$$

In [2] we study the resulting functions and a characterization is also provided.

REFERENCES

[1] C.P. Niculescu *Convexity according to the geometric mean*, Math. Inequal. Appl. no. 2 (2000), 155–167.

[2] P. Jiménez–Rodríguez, M.E. Martínez–Gómez, G.A. Muñoz–Fernández and J.B. Seoane–Sepúlveda *Describing Multiplicative Convex Functions*, Journal of Convex Analysis, prepublished (2019).