THE TAKAGI-VAN DER WAERDEN FUNCTION AND ITS INFINITE DERIVATIVES

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ABSTRACT. Let $r \geq 2$. The Takagi-Van der Waerden function $f_r : [0,1] \to \mathbb{R}$ is defined as follows

$$f_r(x) = \sum_{n=0}^{\infty} \frac{1}{r^n} \phi(r^n x)$$

where $\phi(x)$ denotes the distance from the point x to the nearest integer. These functions are an immediate generalization of the Takagi function and they constitute a family of continuous nowhere differentiable functions. We characterize the set of points where the lateral derivatives of the Takagi-Van der Waerden function are infinite. Furthermore, we determine the Hausdorff dimension and the Lebesgue measure of this set. This is a joint work with J. Ferrera and J. Gómez Gil.

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