ORTHOGONALLY ADDITIVE POLYNOMIALS ON NON-COMMUTATIVE L^p-SPACES

MARÍA LUISA C. GODOY
UNIVERSIDAD DE GRANADA

ABSTRACT. Let $\mathcal{M}$ be a von Neumann algebra with a normal semifinite faithful trace τ. We prove that every continuous m-homogeneous polynomial P from $L^p(\mathcal{M},\tau)$, with $0 < p < \infty$, into each topological linear space X with the property that $P(x+y) = P(x)+P(y)$ whenever x and y are mutually orthogonal positive elements of $L^p(\mathcal{M},\tau)$ can be represented in the form $P(x) = \Phi(x^m)$ ($x \in L^p(\mathcal{M},\tau)$) for some continuous linear map Φ from $L^{p/m}(\mathcal{M},\tau)$ into X.

This result is an extension of the result published by Sundaresan in 1991, which gave a representation for polynomials on $L^p[0,1]$ or ℓ^p, with $1 \leq p < \infty$. Our result is much more general since we not only ensure representation in non-commutative L^p-spaces, but we also contemplate the cases in which the spaces are not Banach spaces.

This is a joint work with J. Alaminos and A. R. Villena.