BOOK OF ABSTRACTS

FUNCTION THEORY ON INFINITE DIMENSIONAL SPACES XV

On occasion of Prof. Jesús Jaramillo's 60th birthday

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Facultad de Ciencias Matemáticas

Universidad Complutense de Madrid

SPAIN

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PLENARY TALKS

Restrictions of surjective functions

Richard Aron. Kent State University. USA

Abstract. The general problem to be considered is the following: Given a "good" *surjective* function $f : X \to Y$, when is there a subspace $Z \subset X$, Z the "same size" as X, such that $f|_Z$ is still surjective? In 2013, Jaramillo, Ransford, and I examined this problem, obtaining mostly negative results. Recently, Jaramillo, Le Donne, and I have looked again at this problem, this time producing mostly positive results.

We will begin by reviewing the two papers cited above. For instance, with Ransford we showed that there is a surjective, $\mathcal{C}^\infty-$ smooth function $f : \ell_2(2^{\aleph_0}) \to \mathbb{R}^2$ whose restriction to any separable subspace of $\ell_2(2^{\aleph_0})$ is no longer surjective. We will then discuss recent, even more negative, results by Hájek and Johanis on the one hand, and even more positive results by Kania and Rmoutil on the other. We include a description of a very recent positive result by Charpentier and Malicet.

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Solid hulls and cores of weighted H^∞-spaces

José Bonet, Instituto Universitario de Matemática Pura y Aplicada IUMPA, Universitat Politècnica de València. Spain

Abstract. We report on joint work with W. Lusky (Paderborn, Germany) and J. Taskinen (Helsinki, Finland).

We determine the solid hull and solid core of weighted Banach spaces H^∞_v of analytic functions functions f such that $v|f|$ is bounded, both in the case of the holomorphic functions on the disc and on the whole complex plane, for a very general class of strictly positive, continuous, radial weights v. Precise results are presented for concrete weights on the disc that could not

be treated before. It is also shown that if H_v^∞ is solid, then the monomials are an (unconditional) basis of the closure of the polynomials in H_v^∞. As a consequence H_v^∞ does not coincide with its solid hull and core in the case of the disc. An example shows that this does not hold for weighted spaces of entire functions. Applications to spaces of multipliers will be mentioned.

Algebraic structures for families of hypercyclic entire functions

J. Alberto Conejero. Universitat Politècnica de València. Spain

`Abstract.` An operator $T \in L(\mathcal{H}(\mathbb{C}))$ is said to be a *convolution operator* provided that it commutes with translations, that is,

$$T \circ \tau_a = \tau_a \circ T \quad \text{for all} \quad a \in \mathbb{C},$$

where $\tau_a f := f(\cdot + a)$. Then $T \in L(\mathcal{H}(\mathbb{C}))$ happens to be a convolution operator if and only if T is an infinite order linear differential operator with constant coefficients $T = \Phi(D)$, where Φ is an entire function with *exponential type*, that is, there exist constants $A, B \in (0, +\infty)$ such that $|\Phi(z)| \leq Ae^{B|z|}$ for all $z \in \mathbb{C}$.

For an entire function of exponential type $\Phi(z) = \sum_{n\geq 0} a_n z^n$ and $f \in \mathcal{H}(\mathbb{C})$, we have

$$\Phi(D)f = \sum_{n=0}^{\infty} a_n f^{(n)}.$$

Godefroy and Shapiro [7] proved that every non-scalar convolution operator is hypercyclic, so covering the classical Birkhoff and MacLane results on hypercyclicity of the translation operator τ_a (take $\Phi(z) = e^{az}$, $a \neq 0$) and of the derivative operator $D : f \mapsto f'$ (take $\Phi(z) = z$), respectively.

Inspired in [1], Bayart and Matheron proved that there is even a residual set of functions in $\mathcal{H}(\mathbb{C})$ generating a hypercyclic algebra for the derivative operator, that is every non-null function in one of these algebras is hypercyclic for the operator D [2, Th. 8.26]. A similar result, with a different approach, was independently obtained by Shkarin [8].

This lead to the following question raised by Aron: *For which functions Φ of exponential type, does $\Phi(D)$ support a hypercyclic algebra?* We provide a partial answer to the above mentioned question by showing the existence of hypercyclic algebras for several convolution operators induced either by polynomials or by transcendental functions [3, 4, 5].

In this line, we also prove the existence of an infinitely generated multiplicative group consisting of entire functions that are, except for the constant function 1, hypercyclic with respect to the convolution operator associated

to a given entire function of exponential type. A certain stability under multiplication is also shown [6].

This is part of joint works with J. Bès, L. Bernal-González, G. Costakis, D. Papathanasiou, and J.B. Seoane-Sepúlveda[1].

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Gradient flows, second order gradient systems and convexity

Aris Daniilidis. Universidad de Chile. Chile

`Abstract.` Based on a connection between the gradient flow of a $\mathcal{C}^2$-smooth function ψ and evanescent orbits of the second order gradient system defined by the square-norm of $\nabla\psi$, under adequate convexity assumption, we obtain the following surprising result for two $\mathcal{C}^2$, convex and bounded from below functions ψ_1, ψ_2:

if $||\nabla\psi_1|| = ||\nabla\psi_2||$, then $\psi_1 = \psi_2 + k$, for some $k \in \mathbb{R}$.

Co-authors: Tahar Boulmezaoud, Philippe Cieutat
(Université de Versailles Saint-Quentin-en-Yvelines)

[1]This research is supported by Junta de Andalucía FQM-127 Grant P08-FQM-03543, GVA, Project ACOMP/2015/005, and MEC Projects: MTM2013-47093-P,MTM2015-65242-C2-1-P, MTM2015-65825-P, and MTM2016-75963-P.

Two extensions of self-expanded maps

Robert Deville. Université de Bourdeaux. France.

Abstract. Our aim is to present a joint work with A. Daniilidis and E. Durand-Cartagena on two generalizations of the notion of self-expanded maps and to study the properties of such maps. Let us recall that if (M, d) be a metric space and I be an interval, a curve $\gamma : I \to M$ is self-expanded if, for all $\tau \in I$, the map $t \mapsto d(\gamma(t), \gamma(\tau))$ is non decreasing on $I \cap [\tau, +\infty)$. If the curve $\gamma : I \to \mathbb{R}^n$ has right derivative - denoted γ' - at each point, then γ is self-expanded if and only if, for all t, τ in I such that $t < \tau$, we have $\langle \gamma'(\tau), \gamma(t) - \gamma(\tau) \rangle \leq 0$. Whenever M is a compact subset of a Riemannian manifold, or whenever M is a compact subset of a finite dimensional vector normed space, all self-expanded maps with values in M have finite length. We introduce two definitions generalizing the notion of self-expanded maps. Given $\lambda \in [0, 1[$ and an interval I, a curve $\gamma : I \to \mathbb{R}^d$ is called a λ-curve if for every $t_1 \leq t_2 \leq t_3$ in I we have

$$d(\gamma(t_1), \gamma(t_2)) \leq d(\gamma(t_1), \gamma(t_3)) + \lambda d(\gamma(t_2), \gamma(t_3))$$

Bounded λ-curves in the euclidean space have finite length whenever $\lambda < 1/d$.

A continuous curve γ, having right derivative at each point, is a λ-eel if, for every $(t, \tau) \in I^2$ such that $t < \tau$,

$$\langle \gamma'(\tau), \gamma(t) - \gamma(\tau) \rangle \leq \lambda \left|\left|\gamma'(\tau)\right|\right| \, ||\gamma(t) - \gamma(\tau)||$$

If $\alpha = \arccos(\lambda)$, $\gamma : I \to \mathbb{R}^n$ is a λ-eel if, for every $\tau \in I$, the open cone $C(\tau, \alpha)$ of origin $\gamma(\tau)$, direction $\gamma'(\tau)$ and of angle α does not meet $\Gamma(\tau)$. A λ-curve is a λ-eel, but the converse is false. Whenever $\lambda \geq \frac{1}{\sqrt{5}}$, we present the construction of a λ-eel γ with values in the unit ball of $\mathbb{R}^3$, and of infinite length. On the other hand, if $\gamma : I \to \mathbb{R}^2$ is a λ-ell for some $\lambda < 1$, and if $\gamma(I)$ is bounded, then the length of γ is finite.

Smooth and Lipschitz approximation of functions defined on metric measure spaces

Rafael Espínola García. Universidad de Sevilla. Spain

Abstract. In this talk we will revise some of the classical results on smooth and Lipschitz approximation of functions defined on $\mathbb{R}^n$ and Banach spaces and will provide a first approach to the problem when the domain of the function is a metric measure space endowed with a measure differentiable

structure (MDS). In particular, we explore density of differentiable functions defined from a metric measure space with a MDS into a Banach space.

New results here presented are part of a joint work with Luis Sánchez González.

Global surjection and inverse theorems

Olivia Gutú. Universidad de Sonora. Mexico

Abstract. We will present a panoramic view on global surjection and inverse mapping theorems for functions between infinite-dimensional spaces. At the end, some pending issues will be discussed.

Representation of unbounded quadratic forms and orthogonal additivity

Juan Manuel Pérez-Pardo. Universidad Carlos III de Madrid. Spain

Abstract. Representation theorems are useful tools in characterizing and dealing with the mathematical objects they are related with. One famous example is the spectral theorem for (unbounded) self-adjoint operators, that allows to work with operators on a Hilbert space as if they were multiplication operators on a space of square integrable functions.

We will consider the case of Hermitian quadratic forms, which are bilinear functions on Hilbert spaces and that constitute another example of maps on a Hilbert space that admit a representation theorem. These representation theorems usually come with an important drawback, the quadratic form needs to be semibounded. There are many important examples where this condition is not met and therefore alternative representation theorems are needed.

We will provide a representation theorem for not-semibounded Hermitian quadratic forms based on a decomposition of direct integral of the Hilbert space and the notion of orthogonal additivity. We will introduce the main ideas in a constructive way and provide meaningful examples.

Some isometric properties of Lipschitz free spaces

Antonin Prochazka. Université Bourgogne Franche-Comté. France

Abstract. The Lipschitz free space $\mathcal{F}(M)$ is a Banach space constructed "around" a given metric space M in such a way that the Lipschitz maps defined on M become linear maps on $\mathcal{F}(M)$. The spectacular results of Godefroy and Kalton in this direction have underlined the usefullness of this concept in the non-linear geometry of Banach spaces. At the same time the norm on $\mathcal{F}(M)$ is closely related to the Wasserstein distance on the probability measures on M known in optimal transport. For these reasons the geometric properties of free spaces became a field of study in itself. In this talk we are going to survey several recent results on the interplay of the geometry of M and the geometry of $\mathcal{F}(M)$. Joint work with L. C. García Lirola, C. Petitjean and A. Rueda Zoca.

Szlenk indices, variations, applications and problems

Matías Raja. Universidad de Murcia. Spain

Abstract. The Szlenk index is powerful tool in isomorphic theory of Banach spaces. We review the relationships between the ordinary Szlenk index, the convex Szlenk index, the dentability index, the optimal bounds between all them and some other recent results. It is well known that the existence of asymptotically uniformly convex dual norms is related to the Szlenk index, however the use of alternative measures of noncompactness gives alike interesting results. We will show that a variation of the Szlenk index provides asymptotically uniformly smooth renormings without appealing to the duality. A few applications to c_0 are given as it is "the most" asymptotically uniformly smooth space. Finally, we will show some applications to the nonlinear classification of Banach spaces.

Injective linearization of dominated polynomials

Pilar Rueda. Universitat de València. Spain

Abstract. Polynomials come naturally in many branches of Mathematics and are particularly useful when trying to approximate, for instance, holomorphic, continuous or integrable functions. In particular, 1-homogeneous

polynomials are linear operators and this is why many people think naively that linear properties can be transferred to polynomials just by means of some induction procedure. Far from being true, we will show the difficulty that arises when dealing with dominated polynomials. Different notions of summing polynomials were introduced as an attempt to generalize the well-known theory of summing linear operators to a non linear context. We will show how to get factorization results through both, L_p spaces and Lorentz spaces, for suitable subclasses of dominated polynomials. An study of these subclasses shows that these polynomials have as injective linearizations the (q, p)-summing operators, and this injective duality will provide connections with integral polynomials and a polynomial version of Pisier's factorization theorem. Some other applications involving cotype of Banach spaces and concavity in Banach lattices are given.

The talk is based in several joint works with G. Botelho, M. Mastylo, D. Pellegrino and E. A. Sánchez Pérez.

What is the Dirichlet problem for functions of least gradient?

Nageswari Shanmugalingam. University of Cincinnati, USA

Abstract. Given a domain and a fixed data for the boundary of the domain (boundary data), the problem finding a function that is p-harmonic in the domain and achieves this prescribed boundary data is called the Dirichlet problem for p-harmonic functions.

For $1 < p < \infty$ we know that such solutions exist if the boundary data is continuous. The case $p = 1$, also called functions of least gradient, is significantly more complicated. If we insist that the solution should be continuous up to the boundary and achieve the prescribed boundary data, such solutions may not exist for a large class of domains. In this talk we will see three different notions of Dirichlet problem, and discuss existence of solutions for those notions. This talk is based on joint works with Riikka Korte, Panu Lahti, Xining Li, Lukas Maly, and Gareth Speight.

Polihedral spaces with symmetric basis

Stanimir Troyanski. Bulgarian Academy of Sciences & Universidad de Murcia. Spain

Abstract. The aim of this talk is to present two results that make the task of finding equivalent polyhedral norms on certain Banach spaces, having

either a an (uncountable) symmetric or unconditional basis, easier and more transparent without referring to the dual space. Some examples of spaces having equivalent polyhedral norms are given.[2]

On the measure of polynomials attaining local máxima on a vertex

Ignacio Zalduendo. Universidad Torcuato di Tella. Argentina

`Abstract.` We calculate the probability that a k-homogeneous polynomial in n variables attain a local maximum on a vertex in terms of the "sharpness" of the vertex, and then study the dependence of this measure on the growth of dimension and degree. We find that the behavior of vertices with orthogonal edges is markedly different to that of sharper vertices. If the degree k grows with the dimension n, the probability that a polynomial attain a local maximum tends to $1/2$, but for orthogonal edges the growth-rate of k must be larger than $n \ln n$, while for sharper vertices a growth-rate larger than $\ln n$ will suffice.

This is joint work with Damián Pinasco.

[2]Research supported by projects MTM2017-86182-P (AEI/FEDER, UE) and MTM2014-54182-P(FEDER).

SHORT TALKS

Mean Li-Yorke chaos in Banach spaces

Antonio Bonilla. Universidad de La Laguna. Spain.

Abstract. We investigate the notion of mean Li-Yorke chaos for operators on Banach spaces. We show that distributional chaos does not imply mean Li-Yorke chaos unlike what happens in the context to topological dynamics on compact metric spaces. We prove that an operator is mean Li-Yorke chaotic if and only if it has an absolutely mean irregular vector. As a consequence, absolutely Cesàro bounded operators are never mean Li-Yorke chaotic. Joint work with N. C. Bernardes Jr and A. Peris.

On a problem by Gurariy concerning subspaces of continuous functions

Hernán Javier Cabana Méndez. Universidad Complutense de Madrid. Spain.

Abstract. Let $A \subseteq \mathbb{R}$ and denote by $\widehat{\mathcal{C}}(A)$ the subset of $\mathcal{C}(A)$ of functions attaining their maximum at a unique point. In 2004 V. I. Gurariy and L. Quarta proved that the set $\widehat{\mathcal{C}}([0,1])\cup\{0\}$ does not contain a 2-dimensional space whereas $\widehat{\mathcal{C}}([0,1))\cup\{0\}$ and $\widehat{\mathcal{C}}(\mathbb{R})\cup\{0\}$ do contain a 2-dimensional space. Using the usual teminology in lineability theory, we can say that $\widehat{\mathcal{C}}([0,1])$ is not 2-lineable whereas $\widehat{\mathcal{C}}(\mathbb{R})$ and $\widehat{\mathcal{C}}([0,1))$ are 2-lineable. During a Nonlinear Analysis Seminar held at Kent State University in the academic year 2003/2004, V. I. Gurariy posed the following question: Is $\widehat{\mathcal{C}}(\mathbb{R})$ (or equivalently $\widehat{\mathcal{C}}([0,1))$) n-lineable for $n \geq 3$? The answer to this question has resisted the efforts of many mathematicians ever since. Using a topological approach based on Moore's Theorem we have been able to prove that $\widehat{\mathcal{C}}(\mathbb{R})$ is not three lineable.
Joint work with G. A. Muñoz-Fernández and J. B. Seoane-Sepúlveda.

Algebrability and D-multiples convolution hypercyclic operators

M. Carmen Calderón Moreno. Universidad de Sevilla. Spain

Abstract. In 1929 Birkoff proved the hypercyclicyty of the translation operator in $H(\mathbb{C})$. Later McLane showed the same for the derivative operator D and, in 1991, Godefroy and Shapiro proved that every non-scalar convolution operato r$\Phi(D)$ is hypercyclic. Since then many authors have obtained properties about the size of the set of hypercyclic elements for $\Phi(D)$, such that, residuality, spaceability, maximal dense-lineability, among others. In this talk we study several sufficient conditions for the existence of algebras of hypercyclic entire functions for D-multiples of convolution operators. This is part of a joint work with L. Bernal-González (University of Sevilla, Spain).

Kalton vs. Rochberg derivation of analytic famlies of Banach spaces

Jesús Castillo. Universidad de Extremadura. Spain.

Abstract. The problem we address in this paper is to what extent the Kalton and Rochberg differential processes commute. To get a clear idea of what is going on, set as sampler the case of the interpolation couple (ℓ_1, ℓ_∞). Fixing $\theta = 1/2$ one gets ℓ_2 as interpolation space, and both its Kalton derived space, denoted dX_θ, as well as its Rochberg derived space, denoted $\mathcal{R}_2 X_\theta$, are the Kalton-Peck space Z_2. But, as we show in this paper, the Z_p-spaces form a new interpolation scale, and we will show that the Kalton derived space dZ_2 of this scale is not even isomorphic to the second Rochberg derived space $\mathcal{R}_4\ell_2$ of the original scale. All this leads to obtain a kind of "dyadic tree" of derived spaces and derivations, which may be all different (which extends and improves previous results of Kalton-Peck and Cabello-Castillo-Kalton) ... or equal. A few applications to stability problems and a negative solution for the correspondence problem for singularity will be presented. Joint work with Willian H. G. Corrêa.

Some new estimates for the *polarization constant* of $L^p(\mu)$ spaces

Marianna Chatzakou. Imperial College London

Abstract. The aim of this work is to use the ideas of functional analysis, interpolation theory and some well known probabilistic arguments to tackle some problems involving symmetric multilinear mappings and homogeneous polynomials on Banach spaces. In particular, for the *polarization constant* of $L^p(\mu)$ spaces we are able to improve some known estimates and to derive some new ones.

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Banach spaces of general Dirichlet series

Yun Sung Choi. POSTECH. Korea

Abstract. We study when the spaces of general Dirichlet series bounded on a half plane are Banach spaces, and show that some of those classes are isometrically isomorphic between themselves. In a precise way, let $\{\lambda_n\}$ be a strictly increasing sequence of positive real numbers such that $\lim_{n\to\infty}\lambda_n = \infty$. We denote by $\mathscr{H}_\infty(\lambda_n)$ the complex normed space of all Dirichlet series $D(s) = \sum_n b_n\lambda_n^{-s}$, which are convergent and bounded on the half plane $[\mathrm{Re}\, s > 0]$, endowed with the norm

$$\|D\|_\infty = \sup_{\mathrm{Re}\, s>0} |D(s)|.$$

If $(*)$ there exists $q > 0$ such that $\inf_n(\lambda_{n+1}^q - \lambda_n^q) > 0$, then $\mathscr{H}_\infty(\lambda_n)$ is a Banach space. Further, if there exists a strictly increasing sequence $\{r_n\}$ of positive numbers such that the sequence $\{\log r_n\}$ is $\mathbb{Q}$-linearly independent, $\mu_n = \boldsymbol{r}^{\boldsymbol{a}}$ for $n = \boldsymbol{p}^{\boldsymbol{a}}$, and $\{\lambda_n\}$ is the increasing rearrangement of the sequence $\{\mu_n\}$, then $\mathscr{H}_\infty(\lambda_n)$ is isometrically isomormorphic to $H_\infty(B_{c_0})$. With this condition $(*)$ we explain more explicitly the optimal cases of the difference among the abscissas $\sigma_c,\ \sigma_b,\ \sigma_u$ and σ_a. Joint work with Un Young Kim and Manuel Maestre.

Subdifferentiable functions satisfy Lusin properties of class C^1 or C^2

Miguel García-Bravo. ICMAT-CSIC-UAM-UC3-UCM. Spain

Abstract. A classical theorem of Lusin states that for every Lebesgue measurable function $f : \mathbb{R}^n \to \mathbb{R}$ and every $\varepsilon > 0$ there exists a continuous function $g : \mathbb{R}^n \to \mathbb{R}$ such that

$$\mathcal{L}^n\left(\{x \in \mathbb{R}^n : f(x) \neq g(x)\}\right) \leq \varepsilon.$$

Several authors have shown that one can take g of class C^k, provided that f has some regularity properties of order k. In this short talk we will present the next new result:

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function. Assume that for a measurable set Ω and almost every $x \in \Omega$ there exists a vector $\xi_x \in \mathbb{R}^n$ such that

$$\liminf_{h\to 0} \frac{f(x+h) - f(x) - \langle \xi_x, h\rangle}{|h|^2} > -\infty.$$

Then we show that f satisfies a Lusin-type property of order 2 in Ω, that is to say, for every $\varepsilon > 0$ there exists a function $g \in C^2(\mathbb{R}^n)$ such that $\mathcal{L}^n(\{x \in \Omega : f(x) \neq g(x)\} \leq \varepsilon$. In particular every function which has a nonempty proximal subdifferential almost everywhere also has the Lusin property of class C^2. We also obtain a similar result (replacing C^2 with C^1) for the Fréchet subdifferential. Finally we provide some examples showing that this kind of results are no longer true for *Taylor subexpansions* of higher order. Joint work with D. Azagra, J. Ferrera and J. Gómez-Gil.

Linear structures and modes of convergence

Pablo José Gerlach Mena. Universidad de Sevilla. Spain

Abstract. Recently, several authors have obtained results about the existence of algebraic structures in certain sequence spaces. For instance, in [1] it is proved the maximal-dense-lineability of sequences convergent to zero in measure but not pointwise almost everywhere in $[0, 1]$. In this talk we are going to present some results in this line. In particular, we focus our attention on sequences in $L_0([0, 1])$ and in $L_0([0, +\infty))$ with several and appropiated modes of convergence. Results presented here are part of a joint work with M.Carmen Calderón-Moreno and José Antonio Prado-Bassas (Universidad de Sevilla).

References

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Banach spaces with relatively weakly open convex combinations of slices of the unit ball

Rainis Haller. University of Tartu. Estonia

`Abstract.` We study Banach spaces which have the CWO property, i.e. every finite convex combination of relatively weakly open subsets of their unit ball is open in the relative weak topology of the unit ball. We show that the CWO property is stable by taking one complemented subspaces. If a Banach space X has a special geometric structure, then for any scattered locally compact Hausdorff space K, the space $C_0(K, X)$ of continuous X-valued functions vanishing at infinity has the CWO property. This is a joint work with T. A. Abrahamsen, J. Becerra Guerrero, V. Lima, and M. Põldvere.

Spaceability in the properties of injectivity and surjectivity

Pablo Jiménez-Rodríguez. Kent State University. USA

`Abstract.` We shall continue the study, started in a paper published in 2016, of the existence of algebraic structures inside of the set of injective linear functions. We will relate such existence with a concept that belongs to the analytic geometry. We will also consider the same problem but focusing on the set of surjective linear functions. In this case, the existence of algebraic structures is related with the search for solutions of linear systems with an infinite number of equations in an infinite number of unknowns. Joint work with Richard Aron, Luis Bernal González, Gustavo Adolfo Muñoz Fernández and Juan Benigno Seoane Sepúlveda.

Converse Taylor theorem and applications: Characterisation of $C^{k,\omega}$-smoothness and Marchaud's theorem

Michal Johanis. Charles University. Czech Republic.

Abstract. We show a uniform version of the converse Taylor theorem. We apply it to show that smoothness with uniformly continuous derivatives can be tested only on restrictions to affine lines. As a further application we obtain Marchaud's theorem in infinte-dimensional spaces. Joint work with Ludek Zajícek.

On absolute sums of Banach spaces preserving average roughness, octahedrality, and strong diameter two properties

Johann Langemets. University of Tartu. Estonia

Abstract. We prove that, if Banach spaces X and Y are δ-average rough, then their direct sum with respect to an absolute norm N is $\delta/N(1,1)$-average rough. In particular, for octahedral X and Y and for p in $(1,\infty)$ the space $X \oplus_p Y$ is $2^{1-1/p}$-average rough, which is in general optimal. Another consequence is that for any δ in $(1,2]$ there is a Banach space which is exactly δ-average rough. We give a complete characterization when an absolute sum of two Banach spaces is octahedral or has the strong diameter 2 property. However, among all of the absolute sums, the diametral strong diameter 2 property is stable only for 1- and ∞-sums. This is a joint work with R. Haller and R. Nadel.

Weighted Takagi Functions

Jesús Llorente. Universidad Complutense de Madrid. Spain

Abstract. The Takagi function was introduced in 1903 by Teiji Takagi as a homely example of a continuous nowhere differentiable function on $[0,1]$. It is defined by

$$\tau(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \mathrm{dist} 2^n x \mathbb{Z}$$

where $\mathrm{dist} x\mathbb{Z}$ is the distance from x to the nearest integer. Generally, it can be defined on $\mathbb{R}$ considering D the set of all dyadic real numbers and a

descomposition of D as an increasing union of the following sets

$$D_n = \left\{ \frac{k}{2^n} : k \in \mathbb{Z} \right\}$$

Then, Takagi function can be defined by

$$T(x) = \sum_{n=1}^{\infty} \text{dist} x D_n$$

where $\text{dist} x D_n$ denotes the distance of the point x to the closed set D_n. An inmediate generalization of the Takagi function is the family of functions on $\mathbb{R}$

$$T_r(x) = \sum_{n=0}^{\infty} \frac{1}{r^n} \text{dist} r^n x \mathbb{Z} \quad r = 2, 3 \ldots$$

Thus, T_2 is the Takagi function and T_{10} was introduced in 1930 by B.L. van der Waerden. For this reason, these functions are named Takagi-van der Waerden functions.

In this short talk, we present a generalization of the Takagi function for any countable dense D subset of $\mathbb{R}$ and a sequence of weights $w = (w_n)_n \in l_\infty$. This generalization is introduced in the following way; we consider $D \subset \mathbb{R}$ as above and a descomposition $\mathcal{D} = \{D_n\}_n$ with the property of being an increasing sequence of discrete subsets of D that satisfies

$$D = \bigcup_n D_n$$

Given a sequence of weights $w \in l_\infty$, we define a generalization of the Takagi function $T_{\mathcal{D},w} : \mathbb{R} \to \mathbb{R}$ associated to the descomposition $\mathcal{D}$ of D as

$$T_{\mathcal{D},w}(x) = \sum_{n=1}^{\infty} w_n g_n(x)$$

where $g_n(x) = \text{dist} x D_n$. Our aim is to exhibit the subdifferential of these functions and their properties when the sequence of weights w belongs to l_1 and when this sequence is not in c_0.

On the class of Sierpiński-Zygmund functions

Elena Martínez Gómez. Universidad Complutense de Madrid. Spain

`Abstract.` W. Sierpiński and A. Zygmund showed (1923) that there exists a function $f\colon \mathbb{R} \to \mathbb{R}$ such that, for any set $Z \subset \mathbb{R}$ of cardinality $\mathfrak{c}$, the restriction $f|_Z$ is not continuous. Functions enjoying this property are called, in the literature, Sierpiński-Zygmund function.

Some of the very interesting properties enjoyed by these functions are related to other (more modern) classes of functions (such as "*everywhere*

surjective" functions, functions "*everywhere like*" fractals, almost continuous functions, among others).

The existence of large structures within the class of Sierpiński–Zygmund functions depends, strongly, on the set theoretical setting we are working on, reaching some undecidability results unless some additional hypothesis are considered. In this short talk we shall provide an overview on what it is known about the class of Sierpiński–Zygmund functions and its topological and algebraic genericity within different set theoretical settings. This is a joint work with Profs. Krzysztof C. Ciesielski, Gustavo A. Muñoz, and Juan B. Seoane.

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Hardy-Sobolev spaces and composition operators

Pedro J. Miana. Universidad de Zaragoza. Spain

`Abstract.` In this talk we introduce a scale of Hardy spaces on the half plane $(H_2^{(n)})_{n\geq 0}$, called Hardy-Sobolev spaces. These spaces are of reproducing kernels and we probe a extended Paley-Wiener theorem. Finally their composition operators on these new spaces are studied in detail. Joint work with José E. Galé, Victor Matache and Luis Sánchez Lajusticia.

Differentiable convex extensions of convex functions

Carlos Mudarra. ICMAT. Spain

`Abstract.` Given an arbitrary subset E of $\mathbb{R}^n$ and two functions $f : E \to \mathbb{R}$, $G : E \to \mathbb{R}^n$, we provide necessary and sufficient conditions on f, G for the existence of a *convex* function F of class $C^1(\mathbb{R}^n)$ such that $F = f$ and $\nabla F = G$ on E. Also, if X is a Hilbert space and ω is a modulus of continuity, we solve a similar problem for convex functions of class $C^{1,\omega}(X)$ by means of an explicit formula. This is a joint work with Daniel Azagra.

Continuity of connected polynomials

Gustavo A. Muñoz-Fernández. Universidad Complutense de Madrid. Spain

Abstract. It is well known that a continuous function $f : \mathbb{R} \to \mathbb{R}$ transforms compact sets into compact sets and connected sets into connected sets. These two results are a part of any standard course on elementary real analysis in one variable. It is equally easy, but not as well known, that if a function $f : \mathbb{R} \to \mathbb{R}$ transforms compact sets into compact sets and connected sets into connected sets, then f is necessarily continuous. In this talk we will show what happens when we drop only one of the previous two conditions that guaranty the continuity of a function $f : \mathbb{R} \to \mathbb{R}$. We will also point out that when we consider the same question for polynomials on an infinite dimensional normed space, the conclusion we arrive at is completely different. In 2010 it was proved that 2 homogeneous connected polynomials are continuous. It was conjectured whether all connected polynomials are continuous. We will see that the conjecture is true for all polynomials on a complex normed space and for polynomials of degree at most two (homogeneous or not) on a real normed space.

Joint work with Hernán Cabana Méndez and Juan B. Seoane Sepúlveda.

Composition operators and frequent hypercyclicity: linear structure

José A. Prado-Bassas. Universidad de Sevilla. Spain

Abstract. In this talk we look for a criterion that guarantees the frequent hypercyclicity of a sequence of composition operators defined on the space of holomorphic functions in a complex domain. Additionally, we are interested on the linear structure of the set of its frequent hypercyclic functions and get that the same criterion allows a large algebraic (and topological) size of this set. Results presented here are part of a joint work with Luis Bernal-González and M.Carmen Calderón-Moreno (Universidad de Sevilla) and Andreas Jung (from Universität Trier).

A hierarchy in the family of real surjective functions

Eva Sáez-Maestro. Universidad Complutense de Madrid. Spain

Abstract. This talk focuses on the study of *extreme surjective* functions in $\mathbb{R}^{\mathbb{R}}$. We present several different types of extreme surjectivity by providing examples and crucial properties. These examples help us to establish a hierarchy within the different classes of surjectivity we deal with. The classes presented here are: everywhere surjective functions, strongly everywhere surjective functions, κ-everywhere surjective functions, perfectly everwhere surjective functions and Jones functions. The algebraic structure of the sets of surjective functions we show here is studied using the concept of lineability. We also reveal unexpected connections between the different degrees of extreme surjectivity given above and other interesting sets of functions such as the space of additive mappings, the class of mappings with a dense graph, the class of Darboux functions and the class of Sierpiński-Zygmund functions in $\mathbb{R}^{\mathbb{R}}$. Joint work with Mar Fenoy, José Luis Gámez and Gustavo A. Muñoz.

Maximal directional derivatives and universal differentiability sets in Carnot groups

Gareth Speight. University of Cincinnati. USA

Abstract. Rademacher's theorem asserts that Lipschitz functions from $\mathbb{R}^n$ to $\mathbb{R}^m$ are differentiable almost everywhere with respect to Lebesgue measure. Such a theorem may not be sharp: if $n > 1$, then there exists a Lebesgue null set N in $\mathbb{R}^n$ containing a point of differentiability for every Lipschitz mapping $f\colon \mathbb{R}^n \to \mathbb{R}$. Such sets are called universal differentiability sets and their construction relies on the fact that existence of an (almost) maximal directional derivative implies differentiability. The techniques involved come from the investigation of Frechet differentiability in infinite dimensional Banach spaces by Preiss. We investigate potential analogues of these results in Carnot groups, which are metric measure spaces admitting translations, dilations, Haar measure, and a Carnot-Caratheodory distance defined by joining points by a constrained family of horizontal curves. We will see that the Euclidean results generalize to Carnot groups with relatively nice geometry (including all step two groups), but in other Carnot groups (such as the Engel group) things may go badly wrong. Based on joint work with Andrea Pinamonti and Enrico Le Donne.

POSTERS

The noncommutative Kalton-Peck spaces

Félix Cabello. Universidad de Extremadura. Spain

Abstract. For every von Neumann algebra M and $0 < p < \infty$ we construct a nontrivial exact sequence of M-bimodules and homomorphisms $0 \to L_p \to Z_p(M) \to L_p \to 0$, where L_p is the Haagerup L_p space built over M. The middle space $Z_p(M)$ can be seen as a noncommutative version of the Kalton-Peck space Z_p.

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Invariant optimal sampling plan in the sequentially planned context

M. Mar Fenoy. Universidad Complutense de Madrid. Spain

Abstract. We define the invariant sequentially planned decision procedure under a group of measurable transformations. By means of a previous reduction of the problem, we obtain the optimal invariant sequentially planned decision procedure that minimizes the risk function, but the solution is based on the past information. Finally, we study the possibility of approximating the general case by a sequence of truncated problems with a maximal number of stages, obtaining a solution based on the present information.

Extending bilinear maps on Banach spaces by homology

Ricardo García. Universidad de Extremadura. Spain

Abstract. Given two Banach spaces $X$and Y let $\mathcal{L}(X, Y)$ denote the vector space of linear continuous operators acting between them; its derived functor is the one that assigns to each couple $X; Y$ the vector space $\mathrm{Ext}(X, Y)$ of exact sequences $0 \to Y \to \square \to X \to 0$ modulo equivalence; let us agree that the second derived functors will be called $\mathrm{Ext}^2(X, Y)$.

Several important Banach space problems and results adopt the form $\mathrm{Ext}(X,Y)=0$ (or $\mathrm{Ext}(X,Y)=0$). For instance,

- Sobczyk's theorem: $\mathrm{Ext}(c_0,X)=0$ for every separable Banach space X.
- Lindenstrauss's lifting principle: $\mathrm{Ext}(L_1(\mu),X)=0$ for every ultrasummand X.
- The Enflo-Lindenstrauss-Pisier and Kalton-Peck construction: $\mathrm{Ext}(\ell_2,\ell_2)\neq 0$.
- The Johnson-Zippin's theorem: $\mathrm{Ext}(H^*,\mathcal{L}_\infty)=0$ for every subspace H of c_0.

In general, a basic Banach space question is whether $\mathrm{Ext}(X,Y)=0$ for a given couple of Banach spaces $X;Y$. Similar questions for Ext^2 have not been treated. Let us write $\mathrm{Ext}^2(X,Y)=0$ to mean that all elements FG of $\mathrm{Ext}^2(X,Y)$ are 0.

Palamodov's Problem 6 in [2] says: Is $\mathrm{Ext}^2(\cdot,E)=0$ for any Fréchet space?
Let us answer it in the negative even in the domain of Banach spaces. Perhaps the most interesting situation is the Hilbert space case:

Problem. Is $\mathrm{Ext}^2(\ell_2,\ell_2)=0$?

for which a few partial results can be obtained. The following unexpected connection wich extension of bilinear forms is proved in [1]:

Theorem. 1. $\mathrm{Ext}^2(\ell_2,\ell_2)=0$ *if and only if whenever* $\ell_1/D_2=\ell_2$, *every bilinear form defined on* D_2 *can be extended to a bilinear form on* ℓ_1.

Joint work with Jesús MF Castillo (University of Extremadura).

This work was supported by projects MTM2016-76958-C2-1-P and IB16056 of Junta de Extremadura.

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Properties of Augé-Hájek-Smith type operators

Sebastián Tapia. Universidad de Chile. Chile

`Abstract.` Let X be a (real or complex) Banach space and R a bounded linear operator on X. We are interested in the orbits of the points $x\in X$

under the action of R, $O_R(x)$. If there exists a point x such that $O_R(x)$ is dense in X, then R is called hypercyclic operator. This is also related to the invariant subset problem which asks if there exists an operator on X with non trivial invariant closed subset. In this work we study a class of operators which refutes Prajiktura's conjecture, which asserted that the set of points whose orbits tend (in norm) to infinity is either empty or dense. The existence of these operators was proved by Hájek and Smith for any separable Banach spaces with a symmetric basis, and was later extended by Augé who constructed such an operator in each separable Banach space.

A Banach-Stone type theorem for Finsler Manifolds

Franciasco Venegas. Universidad de Chile. Chile

`Abstract.` We study the quasi-metric structure of connected and complete Finsler manifolds, by defining a suitable space of smooth functions, and endowing said space with convex and order structure. Then we show that, for X and Y connected and complete Finsler manifolds, a convex order isomorphism T between the spaces $SC_1^1(Y)$ and $SC_1^1(X)$ of 1-semi-Lipschitz and C^1-smooth functions, satisfying $\mathrm{SLip}(T0) < 1$ induces diffeomorphic and almost isometric relations between X and Y, and that T must be of the form $Tf = c \cdot (f \circ \tau) + \phi$, where τ is an isometry, $c > 0$ and $\phi \in SC_1^1(X)$ is a smooth function.

Inscribed and circumscribed polygons that characterize inner product spaces

Diego Yáñez. Universidad de Extremadura. Spain

`Abstract.` Let X be a real normed space with unit sphere S. We prove that X is an inner product space if and only if there exists a real number $\rho = \sqrt{(1+\cos\frac{2k\pi}{2m+1})/2}$, $(k = 1, 2, \ldots, m;\ m = 1, 2, \ldots)$, such that every chord of S that supports ρS touches ρS at its middle point. If this condition holds, then every point $u \in S$ is a vertex of a regular polygon that is inscribed in S and circumscribed about ρS. Joint work with Carlos Benítez Rodríguez and Pedro Martín Jiménez.