Symmetrized function spaces

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Given a Banach or quasi-Banach ideal lattice E of measurable functions on an interval $I = (0, a)$ of the real line (or $I = \mathbb{N}$), its symmetrization $E^{(*)}$ is the set of measurable functions f with decreasing rearrangement belonging to E. Under mild conditions on E, its symmetrization is a quasi-Banach function space. Classical examples of this construction are Lorentz spaces, i.e., symmetrizations of weighted L_p-spaces. More recently studied examples are Lorentz-Orlicz spaces (symmetrizations of weighted Orlicz spaces).

After reviewing some general properties of symmetrized function spaces, I will concentrate on their structure theory. One aspect of this theory consists in determining which ℓ_p spaces they contain isomorphically as subspaces, or as sublattices. It turns out that a simple answer to this question can be given in terms of the original ideal lattice E. In the case of a large class of Lorentz-Orlicz spaces, the set of p's for which there are ℓ_p-sublattices can be identified as the union of three explicit intervals. This unifies classical results going back to the 70's and 80's on Lorentz spaces and Orlicz spaces.

Joint work with Anna H. Kamińska.