

Apéndice

Métodos de integración

RESULTADOS TEÓRICOS

1. TABLA DE PRIMITIVAS INMEDIATAS.

1. $\int x^n dx = \frac{1}{n+1} x^{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x} dx = \log |x|$
3. $\int e^x dx = e^x$
4. $\int a^x dx = \frac{a^x}{\log a}$
5. $\int \operatorname{sen} x dx = -\cos x$
6. $\int \cos x dx = \operatorname{sen} x$
7. $\int \tan x dx = -\log |\cos x|$
8. $\int \cot x dx = \log |\operatorname{sen} x|$
9. $\int \sec x dx = \log |\sec x + \tan x| = \log |\tan (\frac{1}{2}x + \frac{1}{4}\pi)|$
10. $\int \operatorname{csc} x dx = \log |\operatorname{csc} x - \cot x| = \log |\tan \frac{1}{2}x|$
11. $\int \operatorname{arc} \operatorname{sen} \frac{x}{a} dx = x \operatorname{arc} \operatorname{sen} \frac{x}{a} + \sqrt{a^2 - x^2} \quad (a > 0)$
12. $\int \operatorname{arc} \cos \frac{x}{a} dx = x \operatorname{arc} \cos \frac{x}{a} - \sqrt{a^2 - x^2} \quad (a > 0)$
13. $\int \operatorname{arc} \tan \frac{x}{a} dx = x \operatorname{arc} \tan \frac{x}{a} - \frac{a}{2} \log (a^2 + x^2) \quad (a > 0)$

14. $\int \operatorname{sen}^2 mx \, dx = \frac{1}{2m} (mx - \operatorname{sen} mx \cos mx)$
15. $\int \cos^2 mx \, dx = \frac{1}{2m} (mx + \operatorname{sen} mx \cos mx)$
16. $\int \sec^2 x \, dx = \tan x$
17. $\int \csc^2 x \, dx = -\cot x$
18. $\int \operatorname{sen}^n x \, dx = -\frac{\operatorname{sen}^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \operatorname{sen}^{n-2} x \, dx$
19. $\int \cos^n x \, dx = \frac{\cos^{n-1} x \operatorname{sen} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$
20. $\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx \quad (n \neq 1)$
21. $\int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx \quad (n \neq 1)$
22. $\int \sec^n x \, dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx \quad (n \neq 1)$
23. $\int \csc^n x \, dx = -\frac{\cot x \csc^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x \, dx \quad (n \neq 1)$
24. $\int \operatorname{senh} x \, dx = \cosh x$
25. $\int \cosh x \, dx = \operatorname{senh} x$
26. $\int \tanh x \, dx = \log |\cosh x|$
27. $\int \coth x \, dx = \log |\operatorname{senh} x|$
28. $\int \operatorname{sech} x \, dx = \operatorname{arctan}(\operatorname{senh} x)$
29. $\int \operatorname{csch} x \, dx = \log \left| \frac{x}{2} \right| = -\frac{1}{2} \log \frac{\cosh x + 1}{\cosh x - 1}$
30. $\int \operatorname{senh}^2 x \, dx = \frac{1}{4} \operatorname{senh} 2x - \frac{1}{2} x$
31. $\int \cosh^2 x \, dx = \frac{1}{4} \operatorname{senh} 2x + \frac{1}{2} x$
32. $\int \operatorname{sech}^2 x \, dx = \tanh x$
33. $\int \operatorname{senh}^{-1} \frac{x}{a} \, dx = x \operatorname{senh}^{-1} \frac{x}{a} - \sqrt{x^2 + a^2} \quad (a > 0)$
34. $\int \cosh^{-1} \frac{x}{a} \, dx = \begin{cases} x \cosh^{-1} \frac{x}{a} - \sqrt{x^2 - a^2} \\ x \cosh^{-1} \frac{x}{a} + \sqrt{x^2 - a^2} \end{cases} \begin{cases} \left[\cosh^{-1} \left(\frac{x}{a} \right) > 0, a > 0 \right] \\ \left[\cosh^{-1} \left(\frac{x}{a} \right) < 0, a > 0 \right] \end{cases}$
35. $\int \tanh^{-1} \frac{x}{a} \, dx = x \tanh^{-1} \frac{x}{a} + \frac{a}{2} \log |a^2 - x^2|$
36. $\int \frac{1}{\sqrt{a^2 + x^2}} \, dx = \log(x + \sqrt{a^2 + x^2}) = \operatorname{senh}^{-1} \frac{x}{a} \quad (a > 0)$
37. $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \operatorname{arctan} \frac{x}{a} \quad (a > 0)$
38. $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \operatorname{arc} \operatorname{sen} \frac{x}{a} \quad (a > 0)$
39. $\int (a^2 - x^2)^{3/2} \, dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \operatorname{arc} \operatorname{sen} \frac{x}{a} \quad (a > 0)$
40. $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \operatorname{arc} \operatorname{sen} \frac{x}{a} \quad (a > 0)$
41. $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right|$
42. $\int \frac{1}{(a^2 - x^2)^{3/2}} \, dx = \frac{x}{a^2 \sqrt{a^2 - x^2}}$
43. $\int \sqrt{x^2 \pm a^2} \, dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \log |x + \sqrt{x^2 \pm a^2}|$
44. $\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \log |x + \sqrt{x^2 - a^2}| = \cosh^{-1} \frac{x}{a} \quad (a > 0)$
45. $\int \frac{1}{x(a+bx)} \, dx = \frac{1}{a} \log \left| \frac{x}{a+bx} \right|$
46. $\int x \sqrt{a+bx} \, dx = \frac{2(3bx - 2a)(a+bx)^{3/2}}{15b^2}$
47. $\int \frac{\sqrt{a+bx}}{x} \, dx = 2\sqrt{a+bx} + a \int \frac{1}{x \sqrt{a+bx}} \, dx$

48. $\int \frac{x}{\sqrt{a+bx}} dx = \frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$
49. $\int \frac{1}{x\sqrt{a+bx}} dx = \frac{1}{\sqrt{a}} \log \left| \frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}} \right|$ ($a > 0$)
 $= \frac{2}{\sqrt{-a}} \arctan \sqrt{\frac{a+bx}{-a}}$ ($a < 0$)
50. $\int \frac{\sqrt{a^2-x^2}}{x} dx = \sqrt{a^2-x^2} - a \log \left| \frac{a+\sqrt{a^2-x^2}}{x} \right|$
51. $\int x\sqrt{a^2-x^2} dx = -\frac{1}{3}(a^2-x^2)^{3/2}$
52. $\int x^2\sqrt{a^2-x^2} dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8} \operatorname{arcsen} \frac{x}{a}$ ($a > 0$)
53. $\int \frac{1}{x\sqrt{a^2-x^2}} dx = -\frac{1}{a} \log \left| \frac{a+\sqrt{a^2-x^2}}{x} \right|$
54. $\int \frac{x}{\sqrt{a^2-x^2}} dx = -\sqrt{a^2-x^2}$
55. $\int \frac{x^2}{\sqrt{a^2-x^2}} dx = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \operatorname{arcsen} \frac{x}{a}$ ($a > 0$)
56. $\int \frac{\sqrt{x^2+a^2}}{x} dx = \sqrt{x^2+a^2} - a \log \left| \frac{a+\sqrt{x^2+a^2}}{x} \right|$
57. $\int \frac{\sqrt{x^2-a^2}}{x} dx = \sqrt{x^2-a^2} - a \operatorname{arcsen} \frac{a}{|x|}$
 $= \sqrt{x^2-a^2} - a \operatorname{arcsen} \left(\frac{x}{a} \right)$ ($a > 0$)
58. $\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$
59. $\int \frac{1}{x\sqrt{x^2+a^2}} dx = \frac{1}{a} \log \left| \frac{x}{a+\sqrt{x^2+a^2}} \right|$
60. $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \operatorname{arcsen} \frac{a}{|x|}$ ($a > 0$)
61. $\int \frac{1}{x^2\sqrt{x^2 \pm a^2}} dx = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$

62. $\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2}$
63. $\int \frac{1}{ax^2+bx+c} dx = \frac{1}{\sqrt{b^2-4ac}} \log \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right|$ ($b^2 > 4ac$)
 $= \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}}$ ($b^2 < 4ac$)
64. $\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \log |ax^2+bx+c| - \frac{b}{2a} \int \frac{1}{ax^2+bx+c} dx$
65. $\int \frac{1}{\sqrt{ax^2+bx+c}} dx = \frac{1}{\sqrt{a}} \log |2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}|$ ($a > 0$)
 $= \frac{1}{\sqrt{-a}} \operatorname{arcsen} \frac{-2ax-b}{\sqrt{b^2-4ac}}$ ($a < 0$)
66. $\int \sqrt{ax^2+bx+c} dx = \frac{2ax+b}{4a} \sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a} \int \frac{1}{\sqrt{ax^2+b+c}} dx$
67. $\int \frac{x}{\sqrt{ax^2+bx+c}} dx = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a} \int \frac{1}{\sqrt{ax^2+bx+c}} dx$
68. $\int \frac{1}{x\sqrt{ax^2+bx+c}} dx = \frac{-1}{\sqrt{-c}} \log \left| \frac{2\sqrt{c}\sqrt{ax^2+bx+c}+bx+2c}{x} \right|$ ($c > 0$)
 $= \frac{1}{\sqrt{-c}} \operatorname{arcsen} \frac{bx+2c}{|x|\sqrt{b^2-4ac}}$ ($c < 0$)
69. $\int x^3\sqrt{x^2+a^2} dx = (\frac{1}{5}x^2 - \frac{2}{15}a^2)\sqrt{(a^2+x^2)^3}$
70. $\int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \mp \frac{\sqrt{(x^2 \pm a^2)^3}}{3a^2 x^3}$
71. $\int \operatorname{sen} ax \operatorname{sen} bx dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} - \frac{\operatorname{sen}(a+b)x}{2(a+b)}$ ($a^2 \neq b^2$)

$$72. \int \operatorname{sen} ax \cos bx \, dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

$$73. \int \cos ax \cos bx \, dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} + \frac{\operatorname{sen}(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

$$74. \int \sec x \tan x \, dx = \sec x$$

$$75. \int \csc x \cot x \, dx = -\csc x$$

$$76. \int \cos^m x \operatorname{sen}^n x \, dx = \frac{\cos^{m-1} x \operatorname{sen}^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} x \operatorname{sen}^n x \, dx$$

$$= -\frac{\operatorname{sen}^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n} \int \cos^m x \operatorname{sen}^{n-2} x \, dx$$

$$77. \int x^n \operatorname{sen} ax \, dx = -\frac{1}{a} x^n \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$

$$78. \int x^n \cos ax \, dx = \frac{1}{a} x^n \operatorname{sen} ax - \frac{n}{a} \int x^{n-1} \operatorname{sen} ax \, dx$$

$$79. \int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$$

$$80. \int x^n \log ax \, dx = x^{n+1} \left[\frac{\log ax}{n+1} - \frac{1}{(n+1)^2} \right]$$

$$81. \int x^n (\log ax)^m \, dx = \frac{x^{n+1}}{n+1} (\log ax)^m - \frac{m}{n+1} \int x^n (\log ax)^{m-1} \, dx$$

$$82. \int e^{ax} \operatorname{sen} bx \, dx = \frac{e^{ax}(a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2}$$

$$83. \int e^{ax} \cos bx \, dx = \frac{e^{ax}(b \operatorname{sen} bx + a \cos bx)}{a^2 + b^2}$$

$$84. \int \operatorname{sech} x \operatorname{tanh} x \, dx = -\operatorname{sech} x$$

$$85. \int \operatorname{csch} x \operatorname{coth} x \, dx = -\operatorname{csch} x$$

Nota. La función logaritmo neperiano de x la designaremos indistintamente e esta sección por $L x$ o $\log x$.