



## Why Europe needs more Mathematics?

Mathematical thinking and reasoning as well as the resulting mathematical language are major achievements that arose over thousands of years of human culture. Nowadays the efficient use of mathematics is a crucial success factor in almost all innovations in key technologies. While the innovation cycles get shorter, the technologies become more complex. Consequently, flexible mathematical models have to be developed providing the basis to master the arising complexity, to react quickly, and to explore new smart options.

Key technologies and mathematics interact in a mutual and innovation driven process. However, the overwhelming contributions by mathematical research are, typically, invisible and taken for granted. This leads to a significant discrepancy in the support of mathematical research in comparison with the research of key technologies where these mathematical innovations are successfully implemented. Thus, there is the major concern that this lack of recognition ultimately will lead to harmful and system-critical situations. Only a rigorous mathematical analysis of the currently used methods in the development of key technologies will allow a scientifically sound and safe use of these techniques in the future. This includes all methods of artificial intelligence or machine learning, all currently discussed methods of systems based engineering and medical technologies, and all techniques used in strategical decision making in society and economics.

In many application fields classical mathematical methodologies are used with extreme success. In contrast to this, fundamental research in mathematics is often seen as an exclusively intellectual endeavor. However, the impact of this type of research becomes increasingly evident in areas such as digital security, data analytics, or the development of digital twins. Nevertheless, the new developments arising from mathematical research often take a long time to reach direct use in key technologies. This is due to the highly complex process of communicating the newly gained deep insights between mathematics and other research fields.

The basis for maintaining high level research in pure and applied mathematics is mathematics education, i.e., the education of pupils and high school students as well as teachers in mathematics. In contrast to the high importance of education in mathematics we observe that this education is reduced in many European countries with disastrous consequences for the future of the high tech development in Europe. The teaching and learning of mathematical concepts allow to systematically analyze processes, it helps to develop problem solving skills as well as creativity, and it sets the basis for scientific communication. Moreover, as mathematics is a key factor to promote interdisciplinary cooperation in science, technology, and engineering, aspects of applied mathematics need to be incorporated adequately in the mathematics education. The resulting STEAME (Science, Technology, Engineering, Arts, Mathematics, Entrepreneurship) education strategy is to be complemented with appropriate entrepreneurial skills.

Mathematics education research concerns issues of fundamental importance in enhancing and further developing mathematics education at all levels, from kindergarten to university. For example, mathematics education research has produced theories and models of learning and teaching that essentially shape the current European effort to move from passive learning in frontal lecture-types classes to active learning in interactive and technology-enhanced learning environments. These theories and models are attuned to the essence of mathematics as a problem-solving and exploration-based intellectual enterprise. Mathematics education research is especially productive when intuitive pedagogical decisions have proven to be ineffective and when non-trivial ways of teaching and learning should be invented and empirically tested. It also plays a major role in developing and empirical testing of novel modes of life-long professional development of mathematics teachers.

*Conclusion:* To enhance interdisciplinary cooperation and the quick efficient incorporation of new mathematical developments in key technologies, it is essential to establish funding for basic research in mathematics as well as for interdisciplinary cooperation with mathematical research in all high tech funding streams and in the respective evaluation panels. For a successful mathematics education in European schools a major program has to be launched for the education and training of teachers to become facilitators for the teaching and learning of STEAME activities based on the respective research findings in mathematics education.