UNCOUNTABLE BASIC SEQUENCES IN ℓ_∞

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Abstract. We prove, under Continuum Hypothesis, that in ℓ_∞ there exists a subspace X with uncountable basis but without a copy of $\ell_1(\mathbf{c})$

0. Notations and preliminaries. If X is a Banach space, B_X and S_X will be the closed unit ball and the unit sphere of X, respectively, and X^* its topological dual. We denote by w^*-Dens X^* the first cardinal $\mathbf{m}$ such that there exists a set $A \subseteq X^*$ with Card $A = \mathbf{m}$ and $X^* = \overline{[A]}^{w^*}$, i. e. X^* is the w^*-closure of the span of A. Also w^*-Dens B_{X^*} will be the first cardinal $\mathbf{n}$ such that there exists a set $A \subseteq B_{X^*}$ with Card $A = \mathbf{n}$ and $B_{X^*} = \overline{co}^{w^*}(A)$. Let ω_0 be the first infinite ordinal, ω_1 the first uncountable ordinal, $\mathbf{c}$ the cardinal of $\mathbb{R}$ and $\aleph_1$ the cardinal of ω_1. If θ is an ordinal (see [S, pg 589]), a θ-basic sequence in a Banach spaces X is a sequence $\{e_i\}_{i<\theta} \subset X$ such that $\exists K \geq 0$ satisfying that if $i_1 < i_2 < \cdots < i_r < \cdots < i_n < \theta$ and $\{\lambda_k\}_{k=1}^n \subset \mathbb{K}$, then $\|\sum_{k=1}^r \lambda_k e_{i_k}\| \leq K \|\sum_{k=1}^n \lambda_k e_{i_k}\|$. An uncountable basic sequence will be a θ-basic sequence, for some uncountable ordinal θ. If $\{e_i\}_{i<\theta}$ is a θ-basic sequence in some Banach space X, denote by $\overline{[\{e_i\}_{i<\theta}]}$ the closed subspace generated by $\{e_i\}_{i<\theta}$.

1. The problem. It is known that $\ell_1(\mathbf{c})$ embeds isometrically into ℓ_∞. To see this fact, take the quotient $Q : \ell_1 \to C([0,1])$, whose adjoint $Q^* : C([0,1])^* \to \ell_\infty$ is an isometric embedding. As $\ell_1([0,1])$ embeds isometrically into $C([0,1])^*$, we have an isometric copy of $\ell_1([0,1])$ in ℓ_∞.

As ℓ_∞ is very large, one tends to conjecture that in ℓ_∞ there exists uncountable basic sequence completely different from $\ell_1(\omega_1)$, i.e. that there exists a Banach space X with uncountable basis such that w^*-Dens $B_{X^*} = \aleph_0$ but $\ell_1(\omega_1) \nsubseteq X$. If we try to find a such X, we get that the job is very dificult. In fact, the natural candidates to be the example we search -i.e. the Banach spaces X such that $X = Y^*$, with Y separable- do not work. The following (and known) Proposition shows how hard is our task.

1991 *Mathematics Subject Classification.* Primary 46B20.

Key words and phrases. Banach spaces, uncountable basic sequences, ℓ_∞.
Supported in part by DGICYT grant PB 91-0377

Proposition 1. *Let X be a Banach space such that w^*-Dens $B_{X^*} = \aleph_0$, i.e. $X \subseteq \ell_\infty$.*

(I) Assume that $X = Y^*$ with Y separable. Then the following are equivalent: (a) $\ell_1(\omega_1) \subseteq X$; (b) X has an uncountable basic sequence.

(II) Assume that X has an unconditional uncountable basis. Then $\ell_1(\omega_1) \subseteq X$.

(III) Assume that X has a shrinking basis. Then this basis is countable.

Proof.(I) That (a) $\Rightarrow$ (b) is clear. Suppose that (b) holds and that $\ell_1(\omega_1) \nsubseteq X$. By [N, 1.19. Th.] (see also [R, Section 1]) we have that $\ell_1 \nsubseteq Y$. Hence by [Go, PROP. 4] X has the (C) property (see [P] for details about the (C) property). As w^*-Dens $X^* = \aleph_0$, we conclude, by [Gr] (see also [FG]), that X has not uncountable basic sequences. In consequence, (a) holds.

(II) Assume that $\{e_i\}_{i\in I}$ is an unconditional uncountable basis of X. As w^*-Dens $X^* = \aleph_0$, there exists a countable and total set $\{x_n^*\}_{n\geq 1} \subset X^*$. Hence $\exists J \subset I$, with card(J)=card(I), $\exists x_r^*$ and $\exists \epsilon > 0$ such that, $\forall i \in J$, $x_r^*(e_i) \geq \epsilon$ (or $\forall i \in J$, $x_r^*(e_i) \leq -\epsilon$). Thus, by the unconditionallity, we get that $\{e_i\}_{i\in J}$ is equivalent to the canonical basis of $\ell_1(J)$. In particular, if $\{e_i\}_{i\in I}$ is an uncountable symmetric basis of X and w^*- Dens $X^* = \aleph_0$, then $\{e_i\}_{i\in I}$ is equivalent to the canonical basis of $\ell_1(I)$.

(III) If X has a shrinking basis, then X is WCG. Hence Dens $X = w^*$-Dens $X^* = \aleph_0$.■

After this disappointing Prop. 1, we look for an example in the mathematical literature, but we do not find such example. Only we locate some approximations. Here it is two samples.

In [N, pg.1092] Negrepontis constructs the Haydon-Kunen-Talagrand compact. This compact is Hausdorff, non separable (hence in $C(K)$ there exists an uncountable basic sequence) and $\ell_1(\omega_1) \nsubseteq C(K)$. Although w^*-Dens $C(K)^* = \aleph_0, B_{C(K)^*}$ is not w^*-separable. Thus $C(K)$ doesn't embed into ℓ_∞.

Also Talagrand constructs in [T, pg.174] a compact Hausdorff space K such that K is non separable, $C(K)$ does not contain a copy of $\ell_1(\omega_1)$. w^*-Dens $C(K)^* = \aleph_0$ but w^*-Dens $B_{C(K)^*} > \aleph_0$. Thus $C(K)$ doesn't embed into ℓ_∞.

2. The solution. In the light of this facts, we undertake the task of find an example for the above problem. Our construction is contained in the following Theorem:

Theorem 2. *Under continuum hypothesis, there exists a subspace $X \subset \ell_\infty$ with uncountable basis and without a copy of $\ell_1(\omega_1)$.*

Proof. If $\mathcal{Q}$ is a Boolean algebra, there exists, by the Stone Theorem, a unique totally disconnected Hausdorff compact space K such that $\mathcal{Q}$ is isomorphic to the algebra of the clopen (=closed and open) subsets of K. Recall (see [L, pg. 118]) that all Boolean algebra is a $\mathbb{Z}_2$-algebra, over $\mathbb{Z}_2 = \{0,1\}$, and that the Stone compact K associated is the set of the ring homomorphisms $h : \mathcal{Q} \to \mathbb{Z}_2$ with the initial topology for the maps $K \ni h \to h(A), A \in \mathcal{Q}$. Note that we get the same topology if we consider only the maps $h \to h(A)$, with $A \in \mathcal{G} \subset \mathcal{Q}$ and $\mathcal{G}$ a system of generators of $\mathcal{Q}$. Hence we can see K as a part of the compact $\{0,1\}^{\mathcal{G}}$.

By induction, we construct a transfinite sequence $\{\mathcal{Q}_\alpha\}_{\alpha\le\omega_1}$ of algebras of subsets of $\mathbb{N}$ satisfying the following conditions:

(I) If $\alpha < \omega_1, \mathcal{Q}_\alpha$ is countable. If $\alpha \le \beta \le \omega_1, \mathcal{Q}_\alpha \subset \mathcal{Q}_\beta$ and, $\forall B \in \mathcal{Q}_\alpha \backslash\{\emptyset\}$, $\mathrm{card}(B) = \aleph_0$.

(II) If $\beta \le \omega_1$ is a limit ordinal, then $\mathcal{Q}_\beta = \bigcup_{\alpha<\beta} \mathcal{Q}_\alpha$. If $\beta = \alpha + 1, \mathcal{Q}_\beta$ will be the algebra generated adding some suitable $A_\alpha \subset \mathbb{N}$ to the algebra $\mathcal{Q}_\alpha$. Hence $\mathcal{Q}_\beta, \beta \le \omega_1$, will be the algebra generated by $\{A_\alpha\}_{\alpha<\beta}$.

(III) Let K_α be the Stone compact associated to the algebra $\mathcal{Q}_\alpha, \alpha \le \omega_1$. We consider K_α embedded into $\{0,1\}^\alpha$ by mean of the map $K_\alpha \ni h \to \{h(A_\gamma)\}_{\gamma<\alpha}$. Let $p_{\alpha,\beta} : \{0,1\}^\beta \to \{0,1\}^\alpha, \alpha \le \beta$, be the canonical projection, $\varphi_{\alpha,\beta} = p_{\alpha,\beta} \restriction_{K_\beta}$ and $\psi_\alpha : \mathbb{N} \to K_\alpha$ such that $\psi_\alpha(n)$ is the homomorphism evaluation at $n \in \mathbb{N}$. Observe that $\psi_\alpha(\mathbb{N})$ is dense in K_α (hence all K_α are separable), that all the above maps are continuous and that $\psi_\alpha = \varphi_{\alpha,\beta} \circ \psi_\beta,\ \alpha \le \beta \le \omega_1$. If $A \in \mathcal{Q}_\alpha$, then A is a subset of $\mathbb{N}$, whereas $\hat{A}^\beta$ will be the associated clopen of $\mathcal{Q}_\beta, \alpha \le \beta$. Notice that $\hat{A}^\beta = \{h \in K_\beta : h(A) = 1\}$ and that $\varphi_{\alpha,\beta}(\hat{A}^\beta) = \hat{A}^\alpha$, for $\alpha \le \beta \le \omega_1$. If $\alpha < \omega_1$, let $\{L_\alpha^\gamma\}_{\gamma<\omega_1}$ be the family of compact subsets of K_α disjoint from $\psi_\alpha(\mathbb{N})$. For this enumeration we use the continuum hypothesis and that K_α is metric compact, if $\alpha < \omega_1$. Of course, if K_α is finite, i.e. $\mathcal{Q}_\alpha$ is a finite algebra, then $L_\alpha^\gamma = \emptyset, \gamma < \omega_1$, because now $K_\alpha = \psi_\alpha(K_\alpha)$. We will impose that the set $A_\alpha \subset \mathbb{N}$ satisfies: (a) A_α is infinite; (b) $\varphi_{\alpha,\alpha+1}(\hat{A}_\alpha^{\alpha+1}) \cap \varphi_{\gamma,\alpha}^{-1}(L_\gamma^\eta) = \emptyset, \eta, \gamma \le \alpha$; (c) if $B \in \mathcal{Q}_\alpha \backslash \{\emptyset\}$, then $B \cap A_\alpha^c \ne \emptyset$, where $A_\alpha^c = \mathbb{N}\backslash A_\alpha$.

We begin the construction with $\mathcal{Q}_0 = \{\emptyset, \mathbb{N}\}$, whose Stone compact K_0 has only one element. Pick $\beta \le \omega_1$ and assume that we have constructed $\mathcal{Q}_\alpha, \alpha < \beta$, satisfying the above (I), (II) and (III) conditions.

If β is an ordinal limit, put $\mathcal{Q}_\beta = \bigcup_{\alpha<\beta} \mathcal{Q}_\alpha$.

Assume that $\beta = \alpha + 1$. The compact subsets $\varphi_{\gamma,\alpha}^{-1}(L_\gamma^\eta),\ \eta, \gamma \le \alpha$, are disjoint from $\psi_\alpha(\mathbb{N})$. Hence there exists an increasing family of compact sets $\{M_m\}_{m\ge1}$ in K_α such that $M_m \cap \psi_\alpha(\mathbb{N}) = \emptyset$, $\forall m \in \mathbb{N}$, and, for each pair $\eta, \gamma \le \alpha, \exists m = m(\eta, \gamma) \in \mathbb{N}$ satisfying $\varphi_{\gamma,\alpha}^{-1}(L_\gamma^\eta) \subset M_m$. Obviously, if K_α is finite then $M_m = \emptyset, m \ge 1$. For each $m \in \mathbb{N}$, $\exists\{G_{m,p}\}_{p\ge1} \subset \mathcal{Q}_\alpha$ such that $\{1, 2, \dots, p\} \subset G_{m,p}$ and $\hat{G} \cap M_m = \emptyset$.

As $\mathcal{Q}_\alpha$ is countable, assume that $\mathcal{Q}_\alpha = \{B_j\}_{0\le j<\tau}$, with $B_0 = \emptyset$ and $\tau \in \mathbb{N}$ or $\tau = \omega_0$. We choose $p_i, n_{ij} \in \mathbb{N}, i \ge 1, 1 \le j \le 2i, j < \tau$, fulfilling the following requirements:

(a) $p_i < n_{i1} < n_{i2} < \dots < n_{i,2i\wedge2(\tau-1)} < p_{i+1}, i \ge 1$, with $2i \wedge 2(\tau - 1) = \inf\{2i, , 2(\tau - 1)\}$ and $2(\omega_0 - 1) = \omega_0$.

(b) Define $E_{ij} = B_j \cap G_{1,p_1} \cap \dots \cap G_{i,p_i}, 1 \le i, 1 \le j < \tau$. Then we require that $E_{11} \ne \emptyset$ and, if $E_{i-1,j} \ne \emptyset, j \le i, j < \tau$, also that $E_{ij} \ne \emptyset$.

(c) If $1 \le j \le i, j < \tau$, then $n_{i,2j-1} \in B_j$ and, if $E_{ij} \ne \emptyset$, also that $n_{i,2j} \in E_{ij}$.

Here it is a plan for this choice:

(1) Pick $p_1 \in \mathbb{N}$ so that $E_{11} \ne \emptyset$.

(2) Assume that we have chosen $p_i \in \mathbb{N}, i = 1, 2, \ldots, r$, and $n_{ij} \in \mathbb{N}, i = 1, 2, \ldots, r-1, 1 \leq j \leq 2i, j < \tau$, satisfying (1), (2) and (3). Then:
   (a) If $1 \leq j \leq r, j < \tau$, take $n_{r,2j-1}, n_{r,2j}$ satisfying (1), that $n_{r,2j-1} \in B_j$ and, if $E_{rj} \neq \emptyset$, that $n_{r,2j} \in E_{rj}$.
   (b) Finally pick $p_{r+1} > n_{r,2r\wedge 2(\tau-1)}$ so that $E_{r+1,j} \neq \emptyset$, if $E_{rj} \neq \emptyset$ and $1 \leq j \leq r+1, j < \tau$.

The set A_α will be $A_\alpha = \{n_{i,2j} : 1 \leq i, 1 \leq j \leq i, j < \tau, E_{ij} \neq \emptyset\}$ and $\mathcal{Q}_{\alpha+1}$ the algebra generated by $\mathcal{Q}_\alpha$ and A_α. We have:

(α) As $E_{i1} \neq \emptyset$, then $n_{i,2} \in A_\alpha, i \geq 1$, and card $A_\alpha = \aleph_0$. Also $A_\alpha \notin \mathcal{Q}_\alpha$. Indeed, if $\tau > j \geq 1$, $A_\alpha \neq B_j$ because $n_{i,2j-1} \in B_j$ but $n_{i,2j-1} \notin A_\alpha, i \geq j$. Hence $\mathcal{Q}_\alpha \subset \mathcal{Q}_{\alpha+1}$ but $\mathcal{Q}_\alpha \neq \mathcal{Q}_{\alpha+1}$.

(β) $A_\alpha \subset G_{n,p_n}, \forall n \geq 1$. Indeed, if $1 \leq j \leq i < n, j < \tau$, and $n_{i,2j} \in A_\alpha$, then $n_{i,2j} \in G_{n,p_n}$ because $n_{i,2j} < p_n$. If $1 \leq j \leq i \geq n, j < \tau$, and $n_{i,2j} \in A_\alpha$ then $E_{ij} \neq \emptyset$, so that $n_{i,2j} \in E_{ij} \subset G_{1,p_1} \cap \ldots \cap G_{i,p_i} \subset G_{n,p_n}$.

(γ) Card $B_j \cap A_\alpha^c = \aleph_0, 1 \leq j < \tau$, because $n_{i,2j-1} \in B_j \cap A_\alpha^c, i \geq j$. Also card $B_j \cap A_\alpha = \aleph_0, 1 \leq j < \tau$, if $B_j \cap A_\alpha \neq \emptyset$. Indeed, if $B_j \cap A_\alpha \neq \emptyset$, as $A_\alpha \subset G_{n,p_n}, \forall n \geq 1$, we have that $E_{ij} \neq \emptyset, \forall i \geq 1$. Hence $n_{i,2j} \in B_j \cap A_\alpha, \forall i \geq j$. As the sets of $\mathcal{Q}_{\alpha+1}$ are finite unions of sets of the form $B_j \cap A_\alpha$ or $B_j \cap A_\alpha^c$, we get that, $\forall B \in \mathcal{Q}_{\alpha+1} \setminus \{\emptyset\}$, card $B = \aleph_0$.

(δ) Clearly $\varphi_{\alpha,\alpha+1}(\hat{A}_\alpha^{\alpha+1}) \subset \hat{G}_{n,p_n}^\alpha$, because $A_\alpha \subset G_{n,p_n}, \forall n \geq 1$. But $M_n \cap \hat{G}_{n,p_n}^\alpha = \emptyset, n \geq 1$. Hence $\varphi_{\alpha,\alpha+1}(\hat{A}_\alpha^{\alpha+1}) \cap \varphi_{\gamma,\alpha}^{-1}(L_\gamma^\eta) = \emptyset, \gamma, \eta \leq \alpha$.

This end the construction of the transfinite sequence $\{\mathcal{Q}_\alpha\}_{\alpha \leq \omega_1}$. Let see the properties of this construction:

(A) Pick $x \in K_\alpha, \alpha < \omega_1$, and consider x as an element of $\{0,1\}^\alpha$, i.e. $x = (x_i)_{i<\alpha}, x_i = 0$ or 1. Observe that:

$$\{x \in K_\alpha : (x, 1) \in K_{\alpha+1}\} = \varphi_{\alpha,\alpha+1}(\hat{A}_\alpha^{\alpha+1})$$

As $\varphi_{\alpha,\alpha+1}(\hat{A}_\alpha^{\alpha+1}) \cap \varphi_{\gamma,\alpha}^{-1}(L_\gamma^\eta) = \emptyset, \gamma, \eta \leq \alpha$, we get that, if $x \in \bigcup_{\gamma,\eta\leq\alpha} \varphi_{\gamma,\alpha}^{-1}(L_\gamma^\eta)$ and $y \in K_\beta, \alpha \leq \beta$, with $\varphi_{\alpha,\beta}(y) = x$, then $y_i = 0, \alpha \leq i < \beta$.

(B) If $C \subset K_{\omega_1} \setminus \psi_{\omega_1}(\mathbb{N})$ is compact, then $\exists \alpha < \omega_1$ such that $x_i = 0$, for all $x \in C$ and $\alpha \leq i < \omega_1$. Indeed, as C is compact and $\psi_{\omega_1}(m) \notin C, \forall m \geq 1, \exists \{D_n\}_{n\geq 1} \subset \mathcal{Q}_{\omega_1}$ such that, for each $m \in \mathbb{N}$, we can choose a finite subset $\Sigma(m) \subset \mathbb{N}$ so that $C \subset \bigcup_{n\in\Sigma(m)} D_n$ and $m \notin \bigcup_{n\in\Sigma(m)} D_n$. Let $\delta < \omega_1$ be such that $\{D_n\}_{n\geq 1} \subset \mathcal{Q}_\delta$. Then $\varphi_{\delta,\omega_1}(C) \cap \psi_\delta(\mathbb{N}) = \emptyset$ and $\varphi_{\delta,\omega_1}(C) = L_\delta^\gamma$, for some $\gamma < \omega_1$. Take $\alpha = \max\{\gamma, \delta\}$. Then, by (A), if $x \in \varphi_{\delta,\alpha}^{-1}(L_\delta^\gamma)$ and $y \in K_\beta, \alpha \leq \beta \leq \omega_1$, with $\varphi_{\alpha,\beta}(y) = x$, we have that $y_i = 0, \alpha \leq i < \beta$. As $\varphi_{\alpha,\omega_1}(C) \subset \varphi_{\delta,\alpha}^{-1}(L_\delta^\gamma)$, we get the above result. In particular, observe that φ_{β,ω_1} is an homeomorphism for $\alpha \leq \beta \leq \omega_1$.

(C) Card $C(K_{\omega_1})^* = \mathbf{c}$. Indeed, pick a Radon measure λ on K_{ω_1} and decompose $\lambda = \lambda_0 + \sum_{n\geq 1} \lambda_n$, where λ_n is the mass of λ on $\psi_{\omega_1}(n), n \in \mathbb{N}$. From this we get that Card $\left\{\sum_{n\geq 1} \lambda_n : \lambda \in C(K_{\omega_1})^*\right\} \leq \mathbf{c}$. Consider λ_0. There exists a sequence of compact sets $H_n \subset K_{\omega_1} \backslash \psi_{\omega_1}(\mathbb{N})$ such that λ_0 is supported by $H = \bigcup_{n \geq 1} H_n$. By (B), $\exists \alpha < \omega_1$ such that $\varphi_{\alpha,\omega_1}|_H$ is injective. Hence we can see λ_0 as a member of $C(K_\alpha)^*$. But Card $C(K_\alpha)^* = \mathbf{c}$, because K_α is a metric compact space. Thus Card $\{\lambda_0 : \lambda \in C(K_{\omega_1})^*\} \leq \mathbf{c}$. Finally we obtain that Card $C(K_{\omega_1})^* = \mathbf{c}$.

(D) $\ell_1(\omega_1) \nsubseteq C(K_{\omega_1})$. Indeed, if $\ell_1(\omega_1) \subset C(K_{\omega_1})$, then we have a quotient $Q : C(K_{\omega_1})^* \to \ell_\infty(\omega_1)$. As $\ell_1(2^{\omega_1})$ embeds into $\ell_\infty(\omega_1)$, we get that $\ell_1(2^{\omega_1})$ embeds into $C(K_{\omega_1})^*$. But Card $\{\ell_1(2^{\omega_1})\} = 2^{\omega_1} > \mathbf{c}$, a contradiction.

(E) Consider in ℓ_∞ the ω_1-sequence $\{\chi_{A_\alpha}\}_{\alpha<\omega_1}$. We claim that $\{\chi_{A_\alpha}\}_{\alpha<\omega_1}$ is a ω_1-basic sequence. Indeed, take $\alpha_1 < \alpha_2 < \cdots < \alpha_n < \alpha_{n+1} < \omega_1, t_i \in \mathbb{R}, i = 1, 2, \ldots, n+1$, and let be $g = \sum_{i=1}^n t_i.\chi_{A_{\alpha_i}}$. Denote by $\tilde{A}_\alpha$ the set A_α or A_α^c. Then g reach the norm $\| g \|$ in the points of some intersection of the form $B = \bigcap_{i=1}^n \tilde{A}_{\alpha_i}$. Obviously $B \in \mathcal{Q}_{\alpha_n}$ and, therefore, $B \cap A_{\alpha_{n+1}}^c \neq \emptyset$. Pick $y \in B \cap A_{\alpha_{n+1}}^c$. Then:

$$\| g \| = \|g(y)\| = \left\| g(y) + t_{n+1}.\chi_{A_{\alpha_{n+1}}}(y) \right\| \leq \left\| \sum_{i=1}^{n+1} t_i.\chi_{A_{\alpha_i}} \right\|$$

As $\psi_{\omega_1}^* : C(K_{\omega_1}) \to \ell_\infty$, such that $\psi_{\omega_1}^*(u)(n) = u \circ \psi_{\omega_1}(n), n \in \mathbb{N}, u \in C(K_{\omega_1})$, is an isometric isomorphism satisfying that $\psi_{\omega_1}^*(\chi_{\hat{A}_\alpha^{\omega_1}}) = \chi_{A_\alpha}, \alpha < \omega_1$, we conclude that $\overline{[\{\chi_{A_\alpha}\}_{\alpha<\omega_1}]} \subseteq \psi_{\omega_1}^*(C(K_{\omega_1}))$. Hence $\ell_1(\omega_1) \nsubseteq \overline{[\{\chi_{A_\alpha}\}_{\alpha<\omega_1}]}$.

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