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THE DOUBLE TSP WITH MULTIPLE STACKS: A VARIABLE NEIGHBORHOOD SEARCH APPROACH

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Outline

1. Introduction
2. Mathematical Model
3. Neighborhood Structures
4. VNS approach
5. Results and Further Work

The Double TSP with Multiple Stacks (DTSPMS)

- Introduced by Hanne L. Petersen (2006)
  - real life application in Easy Cargo Systems A/S
- Extension of the well known TSP
  - pickups & deliveries → two separated graphs
  - sequencing constraints → multiple stacks
- Each order consists of a pickup location and a delivery location
- Load is organized in several rows in the container
  - different stacks obeying LIFO principle (Last-In-First-Out)

The Double TSP with Multiple Stacks (DTSPMS)

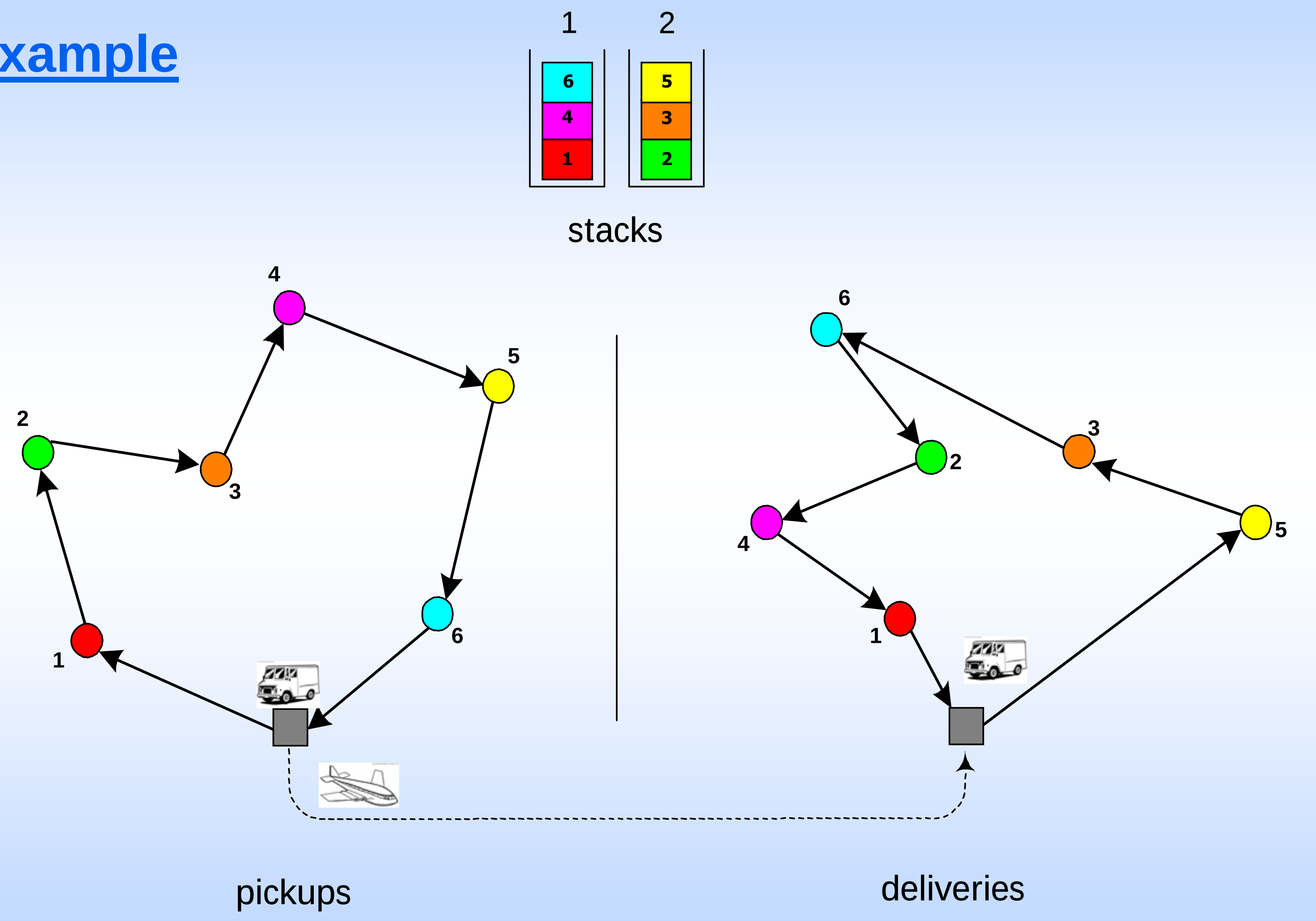
- Pickup & delivery problem with precedence constraints
- Two separated networks, one for pickups and one for deliveries
- All items are uniform
- No repacking allowed
- Transport between the depot of the pickup network and the depot of the delivery network (*longhaul transport*)

  ↳ Not part of the problem

The Double TSP with Multiple Stacks (DTSPMS)

- **Input:**
  - Set of orders (pickup location and delivery location)
- **Output:**
  - pickup route in the first graph
  - delivery route in the second graph
  - loading plan: how to store items
- **Objective:**
  - Minimize the sum of travelled distances

Example



Mathematical Model

$$D = \{1, \cdots, n\}$$ n orders

$$P = \{1, \cdots, m\}$$ m available stacks

maximum capacity: Q

$$N_*^\delta = \{1, \cdots, n\}$$ n nodes in each graph

$$\forall \delta \in \{1, 2\}$$

$$N^\delta = N_*^\delta \cup \{0\}$$

depot

Variables

- **Routing variables:**

$$x_{ij}^{\delta} = \begin{cases} 1 & \text{if } j \text{ follows } i \text{ in route } \delta \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N^{\delta}$$

- **Precedence variables:**

$$y_{ij}^{\delta} = \begin{cases} 1 & \text{if } j \text{ is visited after } i \text{ in route } \delta \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in N_*^{\delta}$$

- **Loading variables:**

$$z_{ip} = \begin{cases} 1 & \text{if order } i \text{ is asigned to stack } p \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in D, \forall p \in P$$

Constraints

- **Flow balance:**

$$\sum_{i \in V^\delta} x_{ij}^\delta = 1 \qquad \forall j \in N^\delta,\ \forall \delta$$

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- **Precedence:**

$$y_{ij}^\delta + y_{ji}^\delta = 1 \qquad \forall i, j \in N_*^\delta,\ i \neq j,\ \forall \delta$$

$$y_{ik}^\delta + y_{kj}^\delta \leq y_{ij}^\delta + 1 \qquad \forall i, j, k \in N_*^\delta,\ \forall \delta$$

$$x_{ij}^\delta \leq y_{ij}^\delta \qquad \forall i, j \in N_*^\delta,\ \forall \delta$$

Constraints

- **LIFO principle:**

$$y_{ij}^{1} + z_{ir} + z_{jr} \leq 3 - y_{ij}^{2} \qquad \forall i, j \in N_{*}^{\delta},\ \forall p \in P$$

- **Loading plan:**

$$\sum_{p \in P} z_{ip} = 1 \qquad \forall i \in D$$

$$\sum_{i \in D} z_{ip} \leq Q \qquad \forall p \in P$$

- **Binary variables:**

$$x, y, z \in \{0, 1\}$$

Objective function

$$\min \sum_{\substack{i,j \in V^\delta \\ \delta \in \{1,2\}}} c_{ij}^\delta \cdot x_{ij}^\delta$$

Minimize the sum of travelled distance in both graphs

Exact solutions

- DTSPMS is a NP-hard problem, more difficult than TSP
- Mathematical model can be solved up to 12 orders in reasonable time

→ **HEURISTICS**

Representation of solutions

- **Before operator**

$$\boxed{S = (\pi_1, \pi_2, \lambda)}$$

$\pi_1 \Rightarrow$ route 1 (pickups) $\pi_2 \Rightarrow$ route 2 (deliveries)

$\lambda \Rightarrow$ loading plan

- **After operator**

$$\boxed{\hat{S} = (\hat{\pi}_1, \hat{\pi}_2, \hat{\lambda})}$$

$\hat{\pi}_1 \Rightarrow$ route 1 (pickups) $\hat{\pi}_2 \Rightarrow$ route 2 (deliveries)

$\hat{\lambda} \Rightarrow$ loading plan

Neighborhood structures

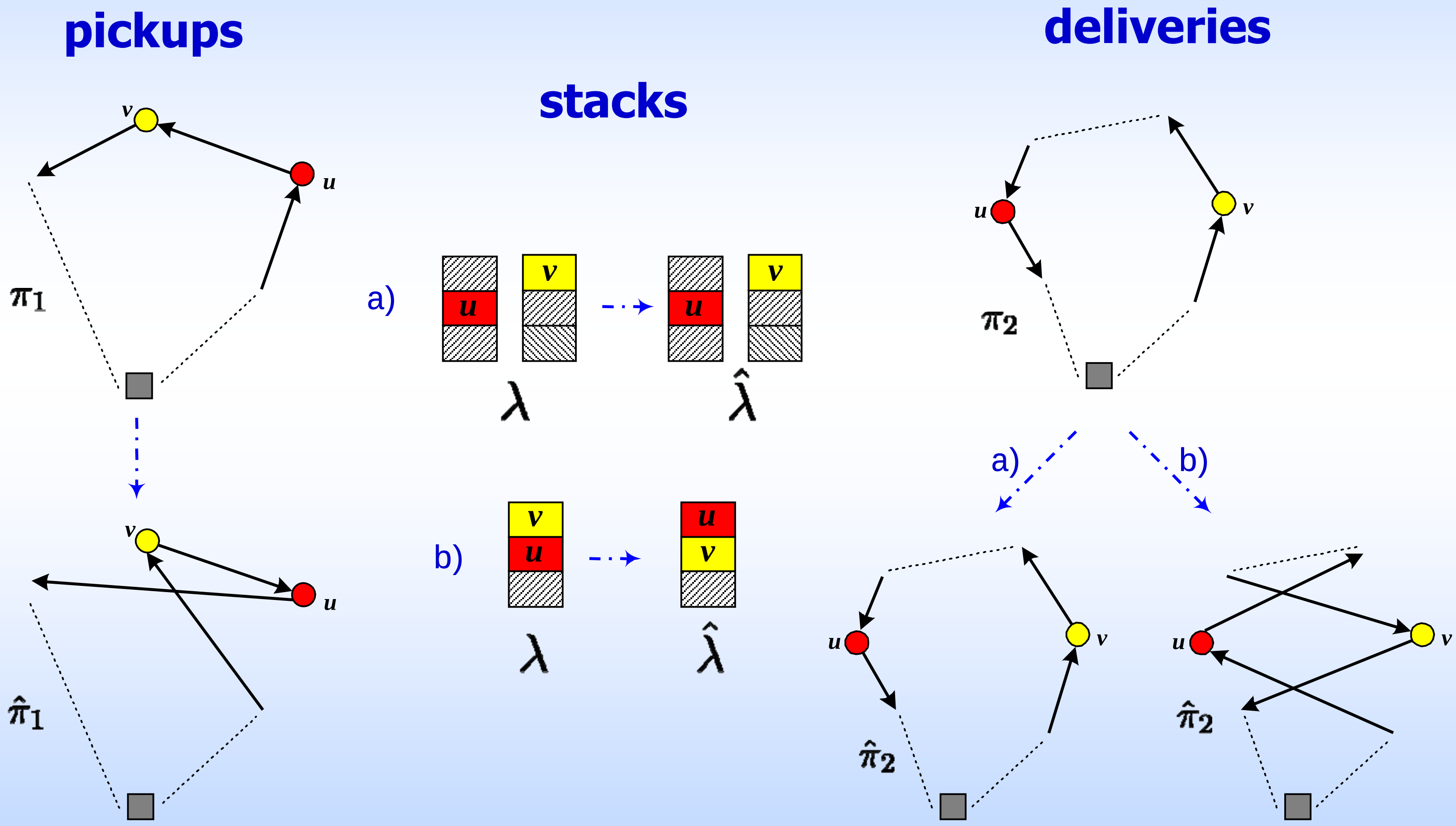
- **Route Swap (RS)**
- **Complete Swap (CS)**

2 operators borrowed from *Hanne L. Petersen (2006)* Tabu Search, Simulated Annealing

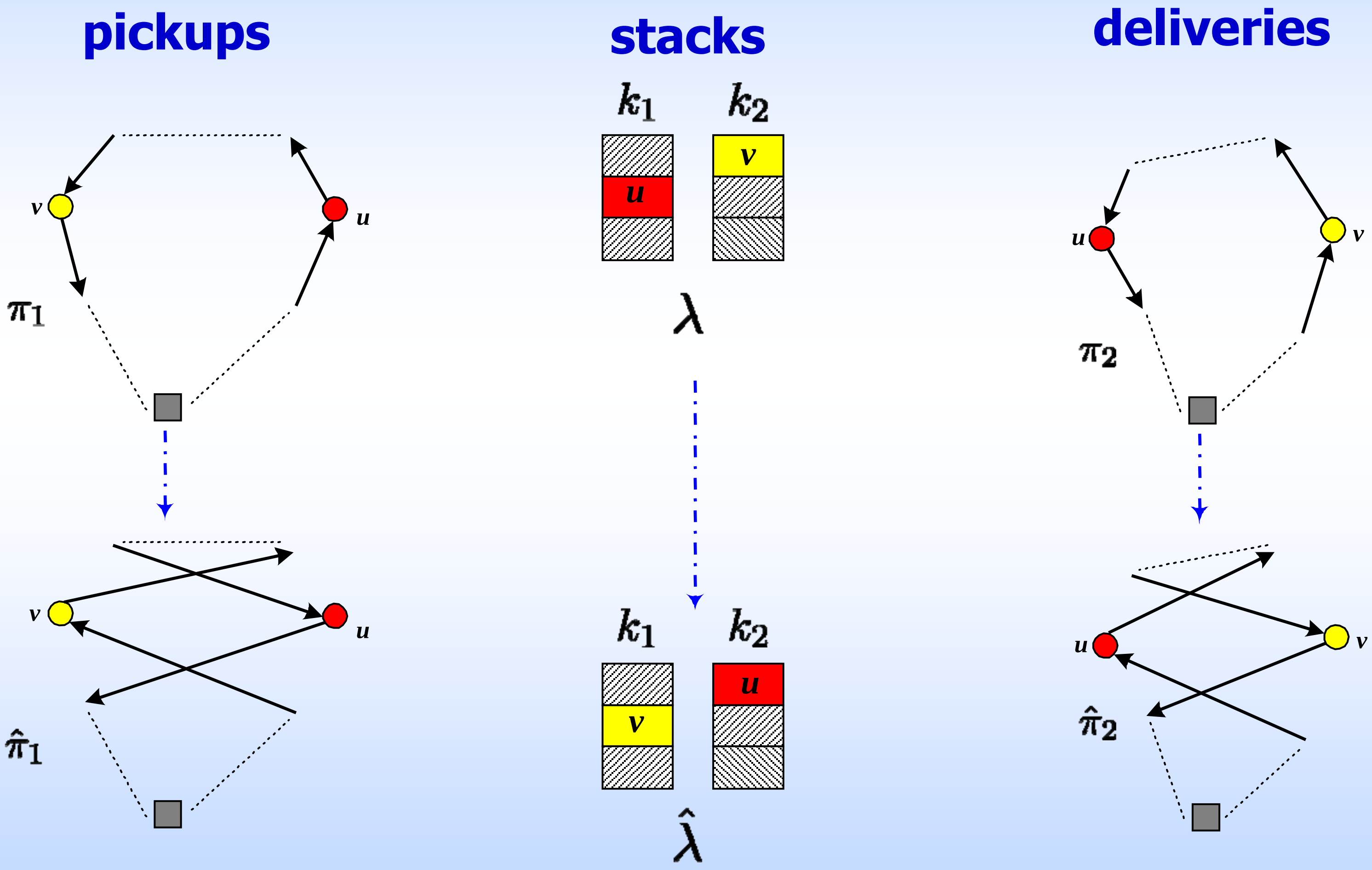
- **In-Stack Swap (ISS)**
- **Reinsertion (R)**
- ***k* – Route Permutation (*k*-RP)**
- ***k* – Stack Permutation (*k*-SP)**

4 new operators

Route Swap

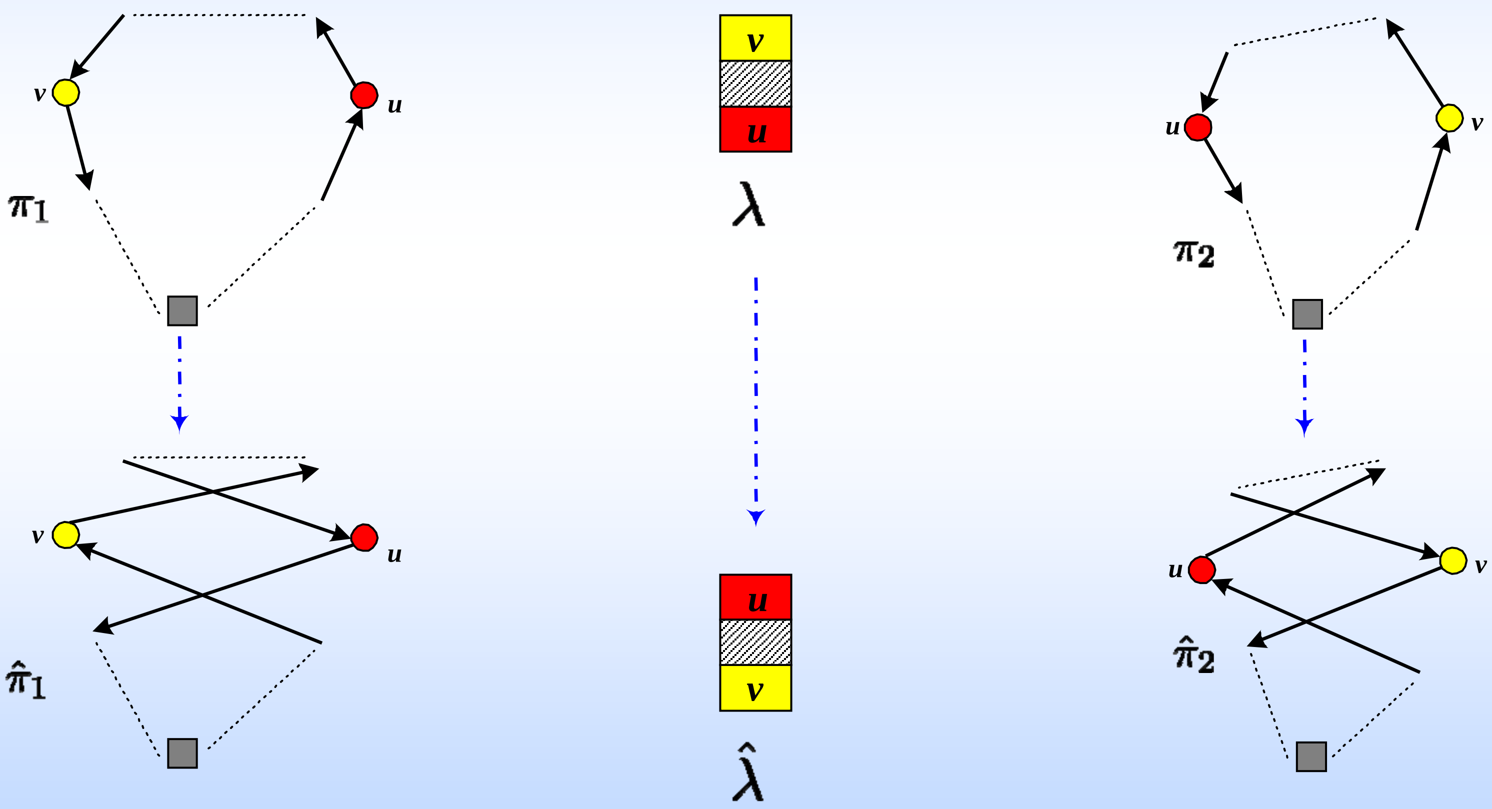

pickups
stacks
deliveries
v
u
π_1
$\hat{\pi}_1$
a)
b)
u
v
λ
$\hat{\lambda}$
π_2
$\hat{\pi}_2$

Complete Swap



In-Stack Swap

pickups **stacks** **deliveries**



Reinsertion

- **Choose:**
  - An order u
  - A stack k
  - A position i^* in route 1
  - A position j^* in route 2

Order u is assigned to stack k and moved to position i^* in route 1 and position j^* in route 2

- Stack k must have room for new items
- Positions i^* and j^* have to be compatible
- The way reinsertion is implemented in route 1 (route 2) depends on the relative order between the position of u in the corresponding route and i^* (j^*).

Reinsertion

- Move order u to position i^* in route 1
- Reassign order u from stack k_1 to k_2
- Reinsertion in route 2 is done the same way

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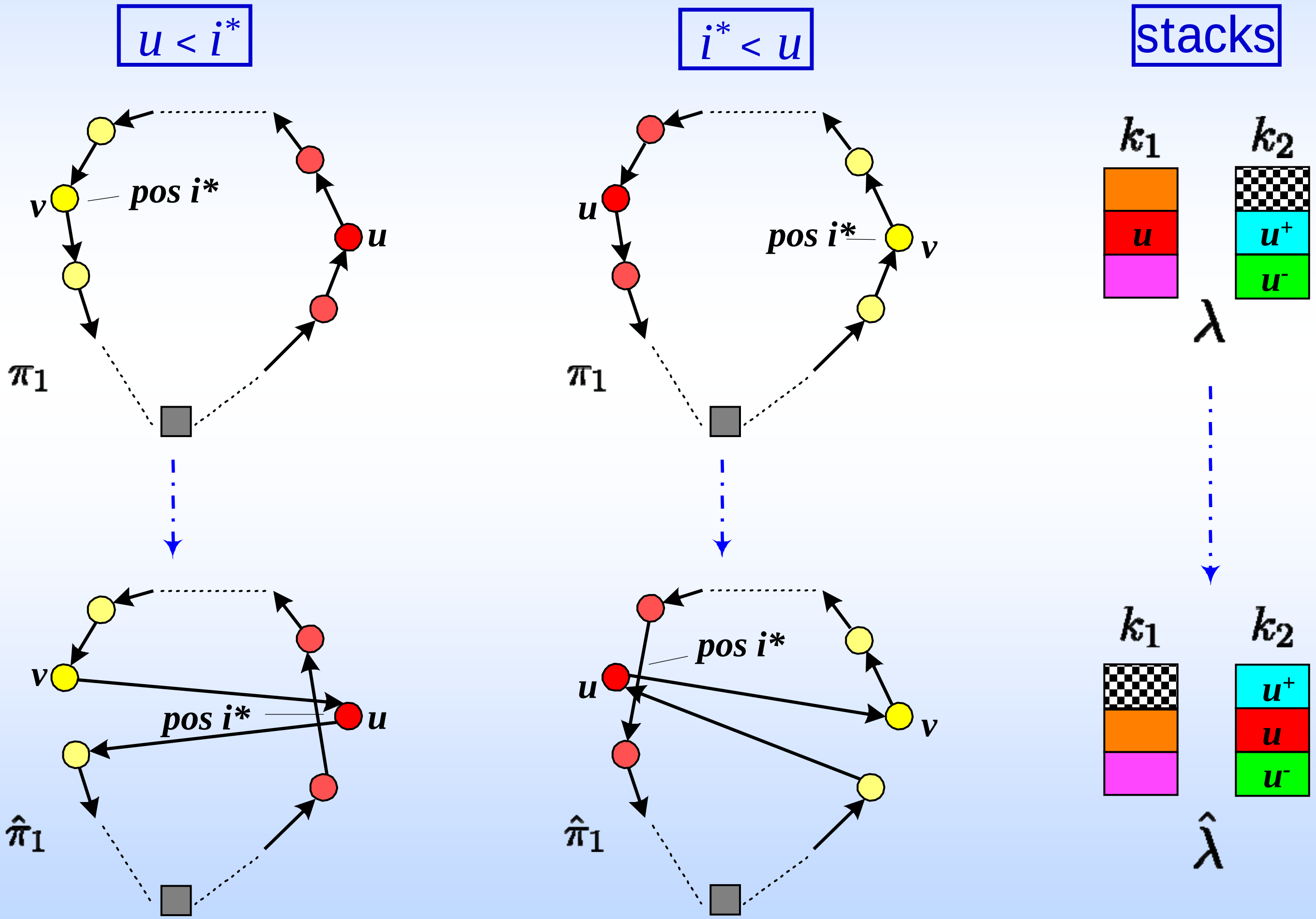
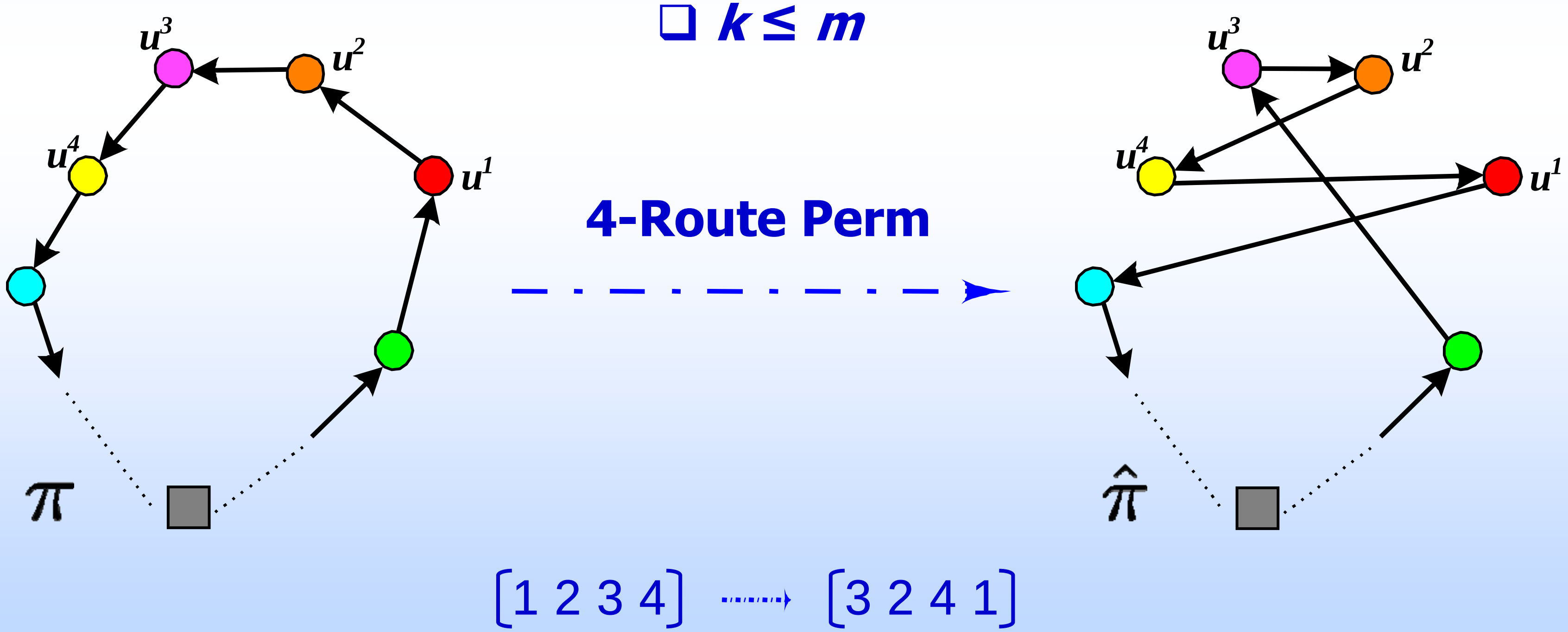


k - Route Permutation

- Choose *k* **consecutive** orders in one route, **assigned to different stacks**, and permute them
- The other route and the loading plan do not change
- $k \leq m$



k - Stack Permutation

- Choose *k* **consecutive** orders in one stack and permute them

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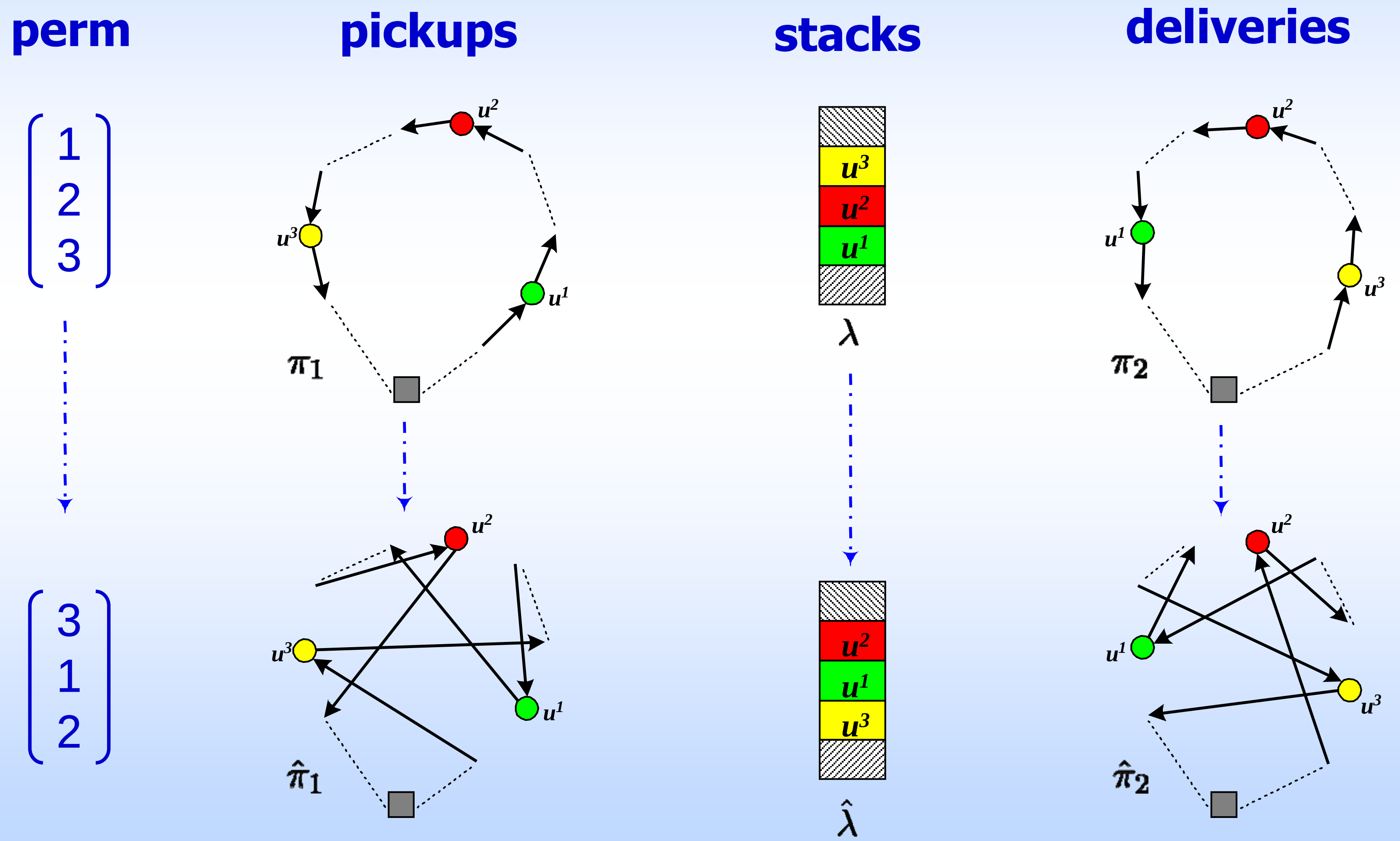

Variable Neighborhood Search (VNS)

- Metaheuristic introduced by Hansen y Mladenovic (1997)
- Main idea: use **different neighborhood structures** and **change between them** when finding a local optimum
- Local optimum with respect to one neighborhood structure is not necessarily local optimum with respect to another one
- Global optimum is local optimum with respect to every neighborhood structure

↳ Using different neighborhood structures usually provides local optima closer to the global optimum

Variable Neighborhood Descent (VND)

1. Initialization:
   1.1. Construct initial solution S
   1.2. $A = \{\Delta_1, \cdots, \Delta_{j_A}\} = \{RS, CS, ISS, R, k\text{-}RP, k\text{-}SP\}$
   1.3. $j \leftarrow 1$
2. Repeat until $j = j_A$:
   2.1. Local Search in Δ_j with initial solution $S \rightsquigarrow S'$
   2.2. If $z(S') < z(S) \implies S = S', j = 1$
   Otherwise $\implies j = j + 1$
3. S is a local minimum with respect to every neighborhood structure in A.

General Variable Neighborhood Search (GVNS)

1. Initialization:
   - 1.1. Construct initial solution S and define A for VND
   - 1.2. $B = \{\Omega_1, \cdots, \Omega_{j_B}\} = \{RS, CS, ISS, R\}$
   - 1.3. $n \leftarrow 1$
2. Repeat until $n = n_{max}$:
   - 2.1. $j \leftarrow 1$
   - 2.2. Perturbation: choose $S' \in \Omega_j(S)$ at random
   - 2.3. Perform VND with S' and $A \rightsquigarrow S''$
   - 2.4. If $z(S'') < z(S) \implies S = S''$, go to step 2.1.
   - 2.5. If $j < j_B \implies j = j + 1$, go to step 2.2.
   - 2.6. $n \leftarrow n + 1$

Test Problems

- Testing has been performed on two sets of 10 randomly generated problems with 33 orders, 3 available stacks with capacity of 11 units.

  Euro pallets

- Best results obtained using Simulated Annealing

(Hanne L. Petersen, 2006)

- Algorithm used: **GVNS**

  - 4 operators for Perturbation: RS, CS, ISS, R
  - 6 operators for VND : RS, CS, ISS, R, *k*-RS, *k*-SS

Set 0

$$\frac{Heur}{Best} \cdot 100$$

			10 seconds		3 minutes	
Problem	LB	Best	SA	GVNS	SA	GVNS
R00	914	1069	25	11.6	12	6.2
R01	875	1072	13	7.6	4	3.8
R02	935	1070	18	9.7	10	8.1
R03	961	1111	21	10.0	11	3.9
R04	933	1090	21	8.7	7	4.6
R05	898	1055	17	4.6	10	3.2
R06	998	1118	18	8.3	10	2.0
R07	962	1118	21	11.1	9	7.7
R08	976	1111	21	12.4	11	9.0
R09	982	1106	13	6.6	6	5.0
average			19	9.1	9	5.3

Set 1

$$\frac{Heur}{Best} \cdot 100$$

			10 seconds		3 minutes	
Problem	LB	Best	SA	GVNS	SA	GVNS
R10	901	1021	23	13.1	14	7.1
R11	892	1040	19	5.1	6	5.3
R12	984	1113	21	10.7	12	4.9
R13	956	1102	21	3.4	6	4.4
R14	879	1059	19	9.4	9	4.9
R15	985	1162	19	7.3	8	4.8
R16	967	1105	19	9.6	9	6.0
R17	946	1096	21	10.5	10	8.1
R18	1008	1180	15	4.5	8	2.1
R19	938	1123	14	6.9	8	3.9
average			19	8.1	9	5.1

Comparison between Operators

GVNS (% using all) − (% removing one)

	Time	All	RS	CS	ISS	R	k-RP	k-SP
Set 0	2 s	10.9%	-1.7	6.7	0.1	17.9	-1.3	0.1
	10 s	9.1%	-0.9	6.4	0.1	14.8	0.1	0.2
Set 1	2 s	9.7%	1.0	6.5	0.7	19.1	0.4	0.2
	10 s	8.1%	0.0	8.7	0.1	18.1	0.0	0.1

- **More operators** $\not\Longrightarrow$ **Better solution**
- **Reinsertion is the most effective neighborhood structure**

Further work

- Generate initial solutions using different methods
- Use different versions of Variable Neighborhood Search algorithms
- Try other metaheuristics: GRASP, Genetic algorithms, Ant Colony Systems, etc.
- Examine exact approaches

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THANK YOU FOR

YOUR ATTENTION

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