Wave Equation with p(x,t) - Laplacian and Damping Term: Existence and Blow-up

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Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with Lipschitz-continuous boundary Γ and $Q_T = \Omega \times (0, T]$. We consider the following boundary value problem

$$\begin{cases} u_{tt} = \operatorname{div}\left(a\,|\nabla u|^{p(x,t)-2}\,\nabla u + \alpha \nabla u_t\right) + b\,|u|^{\sigma(x,t)-2}\,u + f, \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in \Omega;\ u|_{\Gamma_T} = 0,\ \Gamma_T = \partial\Omega. \end{cases} \tag{1}$$

The coefficients $a(x,t),\ \alpha(x,t),\ b(x,t),$ exponents $p(x,t), \sigma(x,t)$ and the source term $f(x,t)$ are given functions of their arguments satisfying

$$0 < a_- \le a(x,t) \le a_+ < \infty,\ 0 < \alpha_- \le \alpha(x,t) \le \alpha_+ < \infty,\ |b(x,t)| \le b_+ < \infty, \tag{2}$$

$$1 < p_- \le p(x,t) \le p_+ < \infty, 1 < \sigma_- \le \sigma(x,t) \le \sigma_+ < \infty, \tag{3}$$

$$f \in L^2(Q_T),\ u_1 \in L^2(\Omega),\ u_0 \in L^2(\Omega) \cap L^{\sigma(\cdot,0)}(\Omega) \cap W^{1,p(\cdot,0)}(\Omega). \tag{4}$$

Problem (1) appears in models of nonlinear viscoelasticity. The local and global existence theorems and blow- up effects for solutions hyperbolic equations of the type (1) with constant exponents of nonlinearity have been studied in many papers , (see, e.g., [5]). However, only papers [6, 7] are devoted to the study of hyperbolic equations of the type (1) with variable nonlinearities. In the present communication, we discuss how the variable character of nonlinearity influences the existence and blow-up theory for the EDPs of the type (1). The analysis is based on the methods developed in [1]-[4].

References

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