On some singular differential equations arising in the modeling of non-elastic shocks in unilateral mechanics and decision sciences

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We consider the second-order differential system with Hessian-driven damping $\ddot{u} + \alpha\dot{u} + \beta\nabla^2\Phi(u)\dot{u} + \nabla\Phi(u) + \nabla\Psi(u) = 0$, where $\mathcal{H}$ is a real Hilbert space, $\Phi, \Psi : \mathcal{H} \to \mathbb{R}$ are scalar potentials, and α, β are positive parameters. A striking property of this system is that, after introduction of an auxiliary variable y, it can be equivalently written as a first-order system involving only the time derivatives $\dot{u}$, $\dot{y}$, and the gradient operators $\nabla\Phi$, $\nabla\Psi$. This allows us to extend the previous analysis to the case of a convex lower semicontinuous function $\Phi : \mathcal{H} \to \mathbb{R} \cup \{+\infty\}$, and so introduce constraints in our model. When $\Phi = \delta_K$ is equal to the indicator function of a closed convex constraint set $K \subseteq \mathcal{H}$, the subdifferential operator $\partial\Phi$ takes account of the contact forces, while $\nabla\Psi$ takes account of the driving forces. In this setting, by playing with the geometrical damping parameter β, we can describe non elastic shock laws with restitution coefficient. Taking advantage of the infinite dimensional framework, we introduce a nonlinear hyperbolic PDE describing a damped oscillating system with obstacle. We complete this study by an asymptotic Lyapunov analysis. We show that this system is dissipative, each trajectory weakly converges to a minimizer of $\Phi + \Psi$, provided that Φ and $\Phi + \Psi$ are convex functions. Exponential stabilization is obtained under strong convexity assumptions. This study opens up interesting perspectives for the modeling of shocks in mechanics and decision sciences (economical shocks, stress and burnout...).

This presentation draws heavily on recent results obtained with P.E. Mainge and P. Redont.

References:

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