Finite Element Solution of Potential Flows Past Sails

A. Bermúdez, R. Rodríguez and M.L. Seoane *

In the last years the competition in nautical sports, as the America's Cup, has been the source of many important developments in mechanical engineering. The computer aided design is the key to the most efficient use of the wind force and to optimize the configuration of the hull and the sails in order to reach a greater speed (see, for instance, [4]).

This work deals with the mathematical and numerical analysis of a simplified two-dimensional model for the interaction between the wind and a sail. The wind is modeled as a steady irrotational plane flow past the sail, satisfying the Kutta-Joukowski condition. This condition guarantees that the flow is not singular at the trailing edge of the sail. The final aim of this research is to develop tools to compute the sail shape under the aerodynamic pressure exerted by the wind. This is the reason why we propose a fictitious domain formulation of the problem (see [3]), involving the wind velocity stream function and a Lagrange multiplier; the latter allows computing the force density exerted by the wind on the sail. Similar to [2], the Kutta-Joukowski condition is imposed in integral form as an additional constraint. The resulting problem is proved to be well posed under mild assumptions. For the numerical solution, we propose a finite element method based on piecewise linear continuous elements to approximate the stream function and piecewise constant ones for the Lagrange multiplier. Error estimates are proved for both quantities and a couple of numerical tests confirming the theoretical results are reported. Finally the method is used to determine the sail shape under the action of the wind. For this purpose, as a first step the sail is modelled as a linear string. The fluid-structure interaction problem is solved by using a segregated iterative algorithm.

References

[1] A. Bermúdez, M.L. Seoane and R. Rodríguez. A fictitious domain method for the numerical two-dimensional simulation of potential flows past sails. *ESAIM: M2AN* (2011) to appear.

[2] J.F. Ciavaldini, M. Pogu and G. Tournemine. Existence and regularity of stream functions for subsonic flows past profiles with sharp trailing edge. *Arch. Rational Mech. Anal.* **93** (1986) 1–14.

[3] V. Girault and R. Glowinski. Error analysis of a fictitious domain method applied to a Dirichlet problem. *Japan J. Indust. Appl. Math.* **12** (1995) 487–514.

[4] N. Parolini and A. Quarteroni. Mathematical models and numerical simulations for the America's Cup. *Comput. Methods Appl. Mech. Engrg.* **194** (2005) 1001–1026.

*Departamento de Matemática Aplicada, Universidade de Santiago de Compostela. Spain (alfredo.bermudez@usc.es, marialuisa.seoane@usc.es). Dep. de Ingeniería Matemática. Universidad de Concepción. Chile (rodolfo@ing-mat.udec.cl).