Nonlinear Models in Partial Differential Equations

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Collected abstract of the all contributions (oral and/or written)

SPEAKERS SESSION

On some singular parabolic equations

Herbert Amann *

We discuss a class of diffusion equations whose diffusion coefficient vanishes on a lower-dimensional submanifold S of the closure of the underlying domain Ω. In the particular case where $S = \partial\Omega$, we are led to study problems without boundary conditions. Such equations generate analytic semigroups possessing maximal regularity in appropriate weighted L_p-Sobolev spaces.

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Wave Equation with p(x,t) - Laplacian and Damping Term: Existence and Blow-up

S. Antontsev *

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with Lipschitz-continuous boundary Γ and $Q_T = \Omega \times (0, T]$. We consider the following boundary value problem

$$\begin{cases} u_{tt} = \operatorname{div}\left(a\,|\nabla u|^{p(x,t)-2}\,\nabla u + \alpha\nabla u_t\right) + b\,|u|^{\sigma(x,t)-2}\,u + f, \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in \Omega;\ u|_{\Gamma_T} = 0,\ \Gamma_T = \partial\Omega. \end{cases} \tag{1}$$

The coefficients $a(x,t),\ \alpha(x,t),\ b(x,t),$ exponents $p(x,t), \sigma(x,t)$ and the source term $f(x,t)$ are given functions of their arguments satisfying

$$0 < a_- \le a(x,t) \le a_+ < \infty,\ 0 < \alpha_- \le \alpha(x,t) \le \alpha_+ < \infty,\ |b(x,t)| \le b_+ < \infty, \tag{2}$$

$$1 < p_- \le p(x,t) \le p_+ < \infty, 1 < \sigma_- \le \sigma(x,t) \le \sigma_+ < \infty, \tag{3}$$

$$f \in L^2(Q_T),\ u_1 \in L^2(\Omega),\ u_0 \in L^2(\Omega) \cap L^{\sigma(\cdot,0)}(\Omega) \cap W^{1,p(\cdot,0)}(\Omega). \tag{4}$$

Problem (1) appears in models of nonlinear viscoelasticity. The local and global existence theorems and blow- up effects for solutions hyperbolic equations of the type (1) with constant exponents of nonlinearity have been studied in many papers , (see, e.g., [5]). However, only papers [6, 7] are devoted to the study of hyperbolic equations of the type (1) with variable nonlinearities. In the present communication, we discuss how the variable character of nonlinearity influences the existence and blow-up theory for the EDPs of the type (1). The analysis is based on the methods developed in [1]-[4].

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On some singular differential equations arising in the modeling of non-elastic shocks in unilateral mechanics and decision sciences

H. Attouch *

We consider the second-order differential system with Hessian-driven damping $\ddot{u} + \alpha\dot{u} + \beta\nabla^2\Phi(u)\dot{u} + \nabla\Phi(u) + \nabla\Psi(u) = 0$, where $\mathcal{H}$ is a real Hilbert space, $\Phi, \Psi : \mathcal{H} \to \mathbb{R}$ are scalar potentials, and α, β are positive parameters. A striking property of this system is that, after introduction of an auxiliary variable y, it can be equivalently written as a first-order system involving only the time derivatives $\dot{u}$, $\dot{y}$, and the gradient operators $\nabla\Phi$, $\nabla\Psi$. This allows us to extend the previous analysis to the case of a convex lower semicontinuous function $\Phi : \mathcal{H} \to \mathbb{R} \cup \{+\infty\}$, and so introduce constraints in our model. When $\Phi = \delta_K$ is equal to the indicator function of a closed convex constraint set $K \subseteq \mathcal{H}$, the subdifferential operator $\partial\Phi$ takes account of the contact forces, while $\nabla\Psi$ takes account of the driving forces. In this setting, by playing with the geometrical damping parameter β, we can describe non elastic shock laws with restitution coefficient. Taking advantage of the infinite dimensional framework, we introduce a nonlinear hyperbolic PDE describing a damped oscillating system with obstacle. We complete this study by an asymptotic Lyapunov analysis. We show that this system is dissipative, each trajectory weakly converges to a minimizer of $\Phi + \Psi$, provided that Φ and $\Phi + \Psi$ are convex functions. Exponential stabilization is obtained under strong convexity assumptions. This study opens up interesting perspectives for the modeling of shocks in mechanics and decision sciences (economical shocks, stress and burnout...).

This presentation draws heavily on recent results obtained with P.E. Mainge and P. Redont.

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Asymptotic behavior of solutions of elliptic and parabolic problems blowing up at the boundary

Catherine Bandle *

Semi linear problems whose nonlinearities grow strongly at infinity give rise to large solutions which blow up at the boundary. The geometry of the boundary has in general no effect to the first order asymptotic behavior of these solutions, which is expressed as a function of the distance to the boundary. It appears only in the higher order terms. In this talk we shall give a survey on the behavior of large solutions near the boundary and discuss some mechanism such as a Hardy potential or additional nonlinear gradient terms, which can have a considerable effect and determine the boundary behavior of the large solutions.

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Finite Element Solution of Potential Flows Past Sails

A. Bermúdez, R. Rodríguez and M.L. Seoane *

In the last years the competition in nautical sports, as the America's Cup, has been the source of many important developments in mechanical engineering. The computer aided design is the key to the most efficient use of the wind force and to optimize the configuration of the hull and the sails in order to reach a greater speed (see, for instance, [4]).

This work deals with the mathematical and numerical analysis of a simplified two-dimensional model for the interaction between the wind and a sail. The wind is modeled as a steady irrotational plane flow past the sail, satisfying the Kutta-Joukowski condition. This condition guarantees that the flow is not singular at the trailing edge of the sail. The final aim of this research is to develop tools to compute the sail shape under the aerodynamic pressure exerted by the wind. This is the reason why we propose a fictitious domain formulation of the problem (see [3]), involving the wind velocity stream function and a Lagrange multiplier; the latter allows computing the force density exerted by the wind on the sail. Similar to [2], the Kutta-Joukowski condition is imposed in integral form as an additional constraint. The resulting problem is proved to be well posed under mild assumptions. For the numerical solution, we propose a finite element method based on piecewise linear continuous elements to approximate the stream function and piecewise constant ones for the Lagrange multiplier. Error estimates are proved for both quantities and a couple of numerical tests confirming the theoretical results are reported. Finally the method is used to determine the sail shape under the action of the wind. For this purpose, as a first step the sail is modelled as a linear string. The fluid-structure interaction problem is solved by using a segregated iterative algorithm.

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Dirichlet problems with singular convection terms and applications with and without Ildefonso

Lucio Boccardo *

In the first hour of the lecture, I will recall some classical results due to G. Stampacchia: let Ω be a bounded, open subset of $\mathbb{R}^N$, $N > 2$ and M be a bounded, elliptic, measurable matrix, $|E| \in L^N(\Omega)$, $f \in L^m(\Omega)$, $m \geq \frac{2N}{N+2}$, $\mu > 0$ large enough, then the boundary value problem

$$(1) \quad -\operatorname{div}(M(x)\nabla u - u\,E(x)) + \mu u = f(x) \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

has a unique weak solution u with some summability properties. If $\mu = 0$, it is possible to prove the coercivity of the differential operator if $\|E\|_{L^N(\Omega)}$ is small enough.

Nevertheless, *in the second hour*, I will show that if $f \in L^m(\Omega)$, $m \geq 1$, existence and summability properties (depending on m, but independent of the size of $\|E\|$) of weak or distributional solutions [L.B.: Boll. UMI 2008, dedicated to G. Stampacchia] can be proved. I will give examples of explicit radial solutions which show how existence and summability results can be lost in the borderline cases when f and E are smooth enough but instead of the assumption $E \in (L^N(\Omega))^N$ the weaker condition $E \in (L^q(\Omega))^N$ for any $q < N$ is imposed.

One of the main parts of the talk (*third hour*) deals with equations with $E \in (L^2(\Omega))^N$. The starting point here is the definition of solution since the distributional definition does not work. It is possible to give meaning to the solution thanks to the concept of *entropy solutions*, introduced some years ago in collaboration with Philippe+Thierry+Juan Luis. Then I wil discuss some existence results about the system

$$\begin{cases} -\operatorname{div}(A(x)\nabla u) + u = -\operatorname{div}(u\,M(x)\nabla z) + f(x) & \text{in } \Omega, \\ -\operatorname{div}(M(x)\nabla z) = u^\theta & \text{in } \Omega, \\ u = z = 0 & \text{on } \partial\Omega. \end{cases}$$

In the last hour, I will recall existence results in collaboration with Ildefonso+Daniela+François for Dirichlet problems of the type

$$(2) \quad -\operatorname{div}(M(x)\nabla u - \Phi(u)) = f(x) \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

and existence results in collaboration with Haim for Dirichlet problems of the type

$$(3) \quad -\operatorname{div}\Big(\frac{M(x)\nabla u}{(1+|u|)^\gamma}\Big) + u = f(x) \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

in order to present the last existence results.

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Jacobians revisited

Haim Brezis*

I will present joint results with H.-M. Nguyen concerning the study of the Jacobian determinant of maps from $\mathbb{R}^N$ into $\mathbb{R}^N$ (and also $\mathbb{S}^N$ into $\mathbb{S}^N$). Surprisingly, we are able to give a robust definition of a Jacobian determinant for a class of maps which do not admit derivatives (for example a Holder condition suffices). New estimates illuminate classical results of Y. Reshetnyak and J.Ball concerning the behavior of the distributional Jacobian under weak convergence in Sobolev spaces

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On the entropy conditions for some flux limited diffusion equations

Vicent Caselles *

In this talk we give a characterization of the notion of entropy solutions of some flux limited diffusion equations for which we can prove that the solution is a function of bounded variation in space and time. This includes the case of the so-called relativistic heat equation and some generalizations. For them we prove that jump set consists in fronts that propagate at the speed given by Rankine-Hugoniot condition and we give on it a geometric characterization of the entropy conditions. Since entropy solutions are functions of bounded variation in space once the initial condition is, to complete this program we study the time regularity of solutions of the relativistic heat equation under some conditions on the initial datum. An analogous result holds for some other related equations without additional assumptions on the initial condition.

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Some asymptotic issues for variational inequalities

Michel Chipot[*]

We would like to study the asymptotic behaviour in $\ell \to +\infty$ of the solution to some elliptic variational inequalities set in cylinders of the type $\ell\omega_1 \mathrm{x} \omega_2$ where ω_1, ω_2 are bounded open subsets of $\mathbf{R^p}$, $\mathbf{R^{n-p}}$ respectively.

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Effective Chemical Processes in Porous Media

Carlos Conca *

This lecture deals with the homogenization of nonlinear models for chemical reactive flows through the exterior of a domain containing periodically distributed reactive solid grains (or reactive obstacles). The mathematical modeling involves diffusion, different types of adsorption rates and chemical reactions which take place at the boundary of the periodic medium

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On nodal domains of u solution to $-\Delta u = \mu u + f$, $u \in H_0^1(\Omega)$ and μ close to an eigenvalue

Jacqueline Fleckinger -in collaboration with J.P.Gossez and F. de Thélin- *

Consider the Dirichlet problem

$$\begin{cases} -\Delta u = \mu u + f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with Ω a smooth bounded domain in $I\!R^N$ and $f \in L^q(\Omega)$, $q > N$. Let $\hat{\lambda}$ be an eigenvalue of $-\Delta$ on $H_0^1(\Omega)$, with $\hat{\varphi}$ an associated eigenfunction. We compare the nodal domains of u with those of $\hat{\varphi}$. In particular, under suitable assumptions on f (in particular $\int_\Omega f\hat{\varphi} > 0$) and on the nodal domain of $\hat{\varphi}$, for μ sufficiently close to $\hat{\lambda}$, then the solution u of the problem has the same number of nodal domains as $\hat{\varphi}$, and the nodal domains of u appear as small deformations of those of $\hat{\varphi}$. But this is not always the case and we exhibit some examples and counterexamples for various hypotheses.

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Apocalypse now: considerations of past and future climate using generalised energy balance models

A. C. Fowler[*]

One of the lessons one learns from the EPICA Antarctic Ice Sheet ice core, which describes climatic history over the last 800,000 years, is that atmospheric CO_2 has fairly faithfully followed proxy (δD) temperatures over the entire period. In turn, this tells us that carbon variation in the atmosphere is a major cause of palaeoclimatic ice ages. In order to understand this, we need to include a description of the carbon cycle. This includes the production of CO_2 by volcanoes, its loss from the atmosphere by acid rain and consequent weathering and run off to the ocean, and the buffering of carbon in the ocean between the reservoirs of bicarbonate, carbonate and dissolved CO_2.

The simplest possible model for the interaction of these components is a compartment (o.d.e.) model for the concentrations of HCO_3^-, CO_3^{2-}, CO_2, as well as calcium ion Ca^{2+}, calcium carbonate $CaCO_3$ and acidity H^+ (pH $= -\log_{10}[H^+]$) in the ocean, together with a conservation law for atmospheric CO_2. To this set we add the simplest (o.d.e.) energy balance model describing the dependence of temperature on cloud and ice albedo, and on CO_2 and cloud greenhouse effect, as well as an ice sheet growth model which allows for nucleation and the ice sheet elevation effect, thus providing for bistability in the growth of the Pleistocene ice sheets.

The simplest asymptotic reduction of this generalised energy balance model is to two equations for ice sheet volume I and $CaCO_3$ concentration N, and in certain circumstance the model exhibits self-sustained oscillations, which represent a candidate for the 100,000 year periodic ice ages of the last half a million years.

In passing, we note the implication of the model for anthropogenic warming: century scale equilibration to an atmosphere where the ice sheets melt, as they are apparently doing, with consequent sea level rise of a metre or more per century.

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Free boundary problems arising in cancer models

Avner Friedmam[*]

Tumor growth can be modeled by a system of PDEs in a the tumor region which is continuously changing in time. The ”free boundary ” of the tumor is held together by cell-to-cell adhesion, which is assumed to be proportional to the mean curvature. The dependent variables are tumor cells densities and nutrient concentration. In addition one needs to provide a constituent law for the tissue, e.g., assuming it to follow Darcy’s law, Stokes equation, etc. In this talk I will describe such models,state existence theorems, show the existence of families of stationary spherical solutions, discuss their stability, and then proceed to describe the existence of non-radially symmetric family of solutions as bifurcation branches of the spherical ones. I will end by stating several open problems

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Some contributions about degenerate parabolic equations

Jacques Giacomoni and Monique Madaune-Tort *

The first part of the talk will trace the history of the relationship and the past and current cooperation between Ildefonso Díaz and the University of Pau.

The second part of the talk will concern the following quasilinear and singular parabolic equation:

$$\begin{cases} u_t - \Delta_p u = \dfrac{1}{u^\delta} + f(x,u) \ \text{ in } \ (0,T)\times\Omega, \\ u = 0 \ \text{ on } (0,T)\times\partial\Omega, \quad u > 0 \text{ in } \ (0,T)\times\Omega, \\ u(0,x) = u_0(x) \ \text{ in } \Omega, \end{cases} \tag{P_t}$$

where Ω is an open bounded domain with smooth boundary in $\mathbb{R}^{\mathrm{N}}$, $1 < p < \infty$, $0 < \delta$ and $T > 0$. We assume that $(x,s) \in \Omega \times \mathbb{R}^+ \to f(x,s)$ is a bounded below Caratheodory function, locally Lipschitz with respect to s uniformly in $x \in \Omega$ and asymptotically sub-homogeneous.

We present in this talk recent results concerning the following issues:

1) existence and uniqueness of weak solutions to (P_t),

2) regularity of weak solutions to (P_t),

3) long-time behaviour and stabilization.

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On a conjecture by C. Sundberg: A numerical investigation

R. Glowinski & A. Quaini *

Carl Sundberg (University of Tennessee-Knoxville) conjectured some time ago that

$$\sup_{\varphi \in E} \frac{\displaystyle\int_0^1 \frac{|\varphi'|^4}{\varphi^6} dx}{1 + \displaystyle\int_0^1 |\varphi''|^2 dx} < +\infty \tag{SI}$$

where

$$E = \{\varphi |\ \varphi \in H^2(0,1),\ \varphi(0) = \varphi(1),\ \varphi'(0) = \varphi'(1),\ \varphi \geq 1\}.$$

Our goal in this lecture is to report on the results of a numerical investigation that has been carried out these last few months in order to verify the veracity of the above Sundberg inequality. Indeed, our numerical experiments strongly suggest that (SI) is true and give also an approximation of the numerical value of the supremum over E of the functional in (SI). A brief description of the numerical methodology used to verify (SI) will be also provided; some of its features are reminiscent of an inverse power method for eigenvalue computations (which is not surprising since the above functional reminds of a Rayleigh quotient).

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New and old problems in Applied Mathematics

Miguel Ángel Herrero*

Ever since its foundation in 1984, the Department of Applied Mathematics of Universidad Complutense has been a meeting point for applied -minded mathematicians with an interest in real -world problems , and eager to become part of the international scientific community, thus breking a tradition of isolation. This trend was fully developed during Ildefonso Diaz's tenure as Head of Department , from 1984 to 1994, and this lecturer particularly benefited from it.

As a short illustration of ongoing research, I will briefly describe recent work on issues as formation of early vasculature, optimization problems in radiotherapy and the search for earliest fossils on Earth, as well as the role of Mathematics in addressing such issues will be discussed.

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Energy Balance Climate Models with Bio-Feedback

Georg Hetzer *

Professor Díaz has made fundamental contributions to the study of energy balance climate models. Some of them serve as starting point for the topic of this talk, incorporating a bio-feedback in a 2-dimensional energy balance climate model.

It is well-known that there is a multitude of interactions between the climate system and the biosphere. E.g., the rise of temperature accompanied by increased rainfall may lead to a richer vegetation which in turn causes an higher absorption of solar radiation and therefore a further increase in temperature. On the other hand, higher temperatures associated with drier conditions may accelerate desertification and lead to a higher albedo with lower average temperatures as consequence.

Energy balance climate models describe the evolution of a long-term mean of temperature by employing the relevant balance equations for the heat fluxes involved. The horizontal heat flux is parameterized by a diffusion operator, and a bio-feedback can, e.g., be introduced by a Volterra map $V = V(u)\phi$ which is, say, the solution family (climate indicator u as parameter, ϕ a fixed initial vegetation) of a two-species competition system. A typical example for the resulting reaction-diffusion problem is

$$\begin{cases} c(x)\partial_t u - \nabla \cdot [k(x)\,|\nabla u|^{p-2}\,\nabla u] + g(u, V(u)\phi)(t)) \\ \qquad\quad \in F(t, x, u, \overline{u}, V(u)\phi)(t)) \qquad t > 0,\ x \in M, \\ \overline{u}(t,x) := \int_{-T}^{0} \beta(s,x) u(t+s,x)\,ds,\ t > 0,\ x \in M, \\ u(s,x) = u_0(s,x), \qquad\qquad\qquad -T \le s \le 0, x \in M. \end{cases}$$

One is interested in nonnegative solutions $u = u(t,x)$ (temperature in Kelvin). M is a closed, compact, oriented Riemannian surface representing the Earth's surface, the positive functions c and k represent the thermal inertia and the diffusivity of the system, respectively, F stands for the absorbed solar radiation flux, and g represents the emitted terrestrial radiation flux.

I plan to discuss some models for V and the basic dynamics of the resulting problem.

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A nonlinear parabolic-hyperbolic system for contact inhibition of cell-growth

Michiel Bertsch,* Danielle Hilhorst,† Hirofumi Izuhara‡ and Masayasu Mimura §

We consider a tumor growth model involving a nonlinear system of partial differential equations which describes the growth of two types of cell population densities with contact inhibition. In one space dimension, it is known that global solutions exist and that they satisfy the so-called *segregation property*: if the two populations are initially segregated - in mathematical terms this translates into disjoint supports of their densities - this property remains true at all later times. In this paper we apply recent results on transport equations and regular Lagrangian flows to obtain similar results in the case of arbitrary space dimension.

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A free boundary problem arising in the analysis of pressure swirl atomizers

Amable Liñán*

Pressure swirl atomizers are used to inject liquid fuels in combustion chambers, in the form of a hollow, conical, high velocity liquid film, so that it penetrates deeply while disintegrating into small droplets. The atomizers have an axisymmetric chamber where the liquid is fed, away from the axis, with an azimuthal circulating velocity; the swirling liquid flows out of the injector chamber through a cylindrical orifice of radius R_I and length L_I, small compared with those of the chamber.

It is possible to show that the axisymmetric flow can be described using an irrotational solution of the inviscid, Euler, equations, with a velocity field derived from a potential that satisfies the Laplace equation. The axisymmetric liquid domain is hollow inside and also outside the injector chamber, limited by two free boundaries that separate the liquid from the gas of the combustion chamber, of negligible density compared with that of the liquid.

We formulate the problem of determining, in terms of the volumetric flow rate and the circulating velocity, the two free boundaries of the liquid fuel, and the main characteristics of the solution, which are given explicitely for moderately large values of the ratio $\dfrac{\mathrm{L}_I}{\mathrm{R}_I}$.

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Traveling waves in a sawtoothed cylinder and their homogenization limit

Hiroshi Matano*

My talk is concerned with a curvature-dependent motion of plane curves in a two-dimensional cylinder with spatially undulating boundary. In other words, the boundary has many bumps and we assume that the bumps are aligned in a spatially recurrent manner.

The goal is to study how the average speed of the traveling wave depends on the geometry of the domain boundary. More specifically, we consider the homogenization problem as the boundary undulation becomes finer and finer, and determine the homogenization limit of the average speed and the limit profile of the traveling waves. Quite surprisingly, this homogenized speed depends only on the maximal opening angles of the domain boundary and no other geometrical features are relevant.

Next we consider the special case where the boundary undulation is quasi-periodic with m independent frequencies. We show that the rate of convergence to the homogenization limit depends on this number m.

This is joint work with Bendong Lou and Ken-Ichi Nakamura.

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An example of functional which is weakly lower semicontinuous on $W_0^{1,p}$ for every $p > 2$ but not on H_0^1

François Murat *

In this work in collaboration with Fernando Farroni and Raffaella Giova, we give an example of functional which is defined and coercive on $H_0^1(\Omega)$, which is sequentially weakly lower semicontinuous on $W_0^{1,p}(\Omega)$ for every $p > 2$, but which is not sequentially lower semicontinuous on $H_0^1(\Omega)$. This functional is non local.

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A parabolic-elliptic system of drift-diffusion type with subcritical mass in $\mathbb{R}^2$

Toshitaka Nagai *

We consider the Cauchy problem to the following parabolic-elliptic system in $\mathbb{R}^2$:

$$\text{(CP)} \quad \begin{cases} \partial_t u = \Delta u - \nabla \cdot (u \nabla \psi), & t > 0,\ x \in \mathbb{R}^2, \\ -\Delta \psi = u, & t > 0,\ x \in \mathbb{R}^2, \\ u(0, x) = u_0(x), & x \in \mathbb{R}^2. \end{cases}$$

Here ψ is specified as

$$\psi(t, x) = \frac{1}{2\pi} \int_{\mathbb{R}^2} \log \frac{1}{|x - y|} u(t, y)\, dy,$$

and $u(t, x)$ and $u_0(x)$ are nonnegative. This system is a simplified version of chemotaxis system derived from the original Keller-Segel model in $\mathbb{R}^2$, and also a model of self-attracting particles in $\mathbb{R}^2$.

The total mass of the nonnegative solution u to (CP) is conserved, namely $\int_{\mathbb{R}^2} u(t, x)\, dx = \int_{\mathbb{R}^2} u_0(x)\, dx$, and the global existence of the solution heavily depends on the total mass $\int_{\mathbb{R}^2} u_0(x)\, dx$. It is known that the nonnegative solution may blowup in finite time in the supercritical case $\int_{\mathbb{R}^2} u_0(x)\, dx > 8\pi$. In this talk, we consider the subcritical case $\int_{\mathbb{R}^2} u_0(x)\, dx < 8\pi$ and discuss the global existence and large time behavior of the nonnegative solution.

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A variational approach to Navier-Stokes

Pablo Pedregal *

We introduce a variational approach to treat the Navier-Stokes equations both in dimensions 2 and 3. Though the method allows the full treatment in dimension 2, we seek to precisely stress where it breaks down for dimension 3. The basic feature of the procedure is to look directly for strong solutions, by minimizing a suitable error functional that measures the departure of feasible fields from being a solution of the problem. By considering the divergence-free property as part of feasibility, we are able to avoid the explicit analysis of the pressure. Two main points in our analysis are:
1. Coercivity for the error functional is achieved by looking at scaling.
2. Zero is the only critical value: global minimizers of the error are shown to have zero error (and thus they are solutions of the problem) by looking at optimality conditions, which lead to investigate the linearized problem.

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Optimal shapes with convexity constraint

Michel PIERRE *

We will discuss some questions arising when optimizing shapes in the class of convex bodies. Since a convexity constraint provides compactness properties, it is generally easy to prove existence of optimal shapes in this class. On the other hand, the qualitative study of optimal shapes is hard, due to the difficulty of writing the Euler-Lagrange equation. Extracting information from optimality conditions is a serious issue. We will give some recent results concerning convex optimal shapes in two dimensions where it is possible to analytically write complete first and second order optimality conditions. We may deduce a description of a subclass of problems for which optimal shapes are always polygons, and another class for which no corner can appear at the boundary. Other situations surprising lead to $C^{1,1/2}$- regularity at the junction between flat and stricrly convex parts of the boundary.

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The Partial Differential Equations of Finance

Olivier Pironneau*

The object of financial mathematics is to price financial object and evaluate the risks.

To minimize risks self financing portfolios has been a great source of models. Very simple - yet unrealistic - assumptions such as brownian randomness and market completeness (no arbitrage) lead to stochastic differential equations and Itô calculus then gives the partial differential equations for pricing the financial objects.

Even in this restricted setting a large class of partial differential equations can be generated, mostly parabolic in nature, linear but with non constant coefficients. Variational methods work in a slightly adapted Hilbert space setting (weighted Sobolev spaces).

At first one dimensional in space, the partial differential equations are now multidimentional because of stochastic volatility models or because of stochastic interest rates or because the object depends on several assets. For these existence is not always guaranteed, the so called Feller condition is necessary in the case of Stein-Stein models. They can also be degenerate as in the case of Asian options.

New boundary conditions are needed sometimes such a for Bermuda options.

Non-linear systems are not used often so far; but an important class of objects do need nonlinear models, when early exercise is allowed such as for American options. Then the problem is best solved as a variational inequality.

When the Brownian assumption is relaxed, like when jump processes are added, a partial integro differential system is obtained, similar to the equations for radiative transfers in engineering. In several dimensions these problems are challenging and not solved yet.

Finally options on compound objects or basket options lead to partial differential equations in many dimensions as much as 50 or more. These are numerically extremely challenging.

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Fujita phenomenon in the hyperbolic space

Maria Assunta Pozio*

Since the pioneering work of Fujita in 1966, it is known that the Cauchy problem for reaction diffusion equations with power nonlinearities exhibits blow-up, but whether this is the case for all solutions, depends on a critical value, the Fujita exponent. We will discuss here similar results which hold in the hyperbolic space and were obtained with Catherine Bandle and Alberto Tesei. Such results show that the hyperbolic space is closer to a bounded domain, then to the whole Euclidean space.

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Stellerator model and some properties of the relative rearrangement

Jean-Michel Rakotoson *

The introduction of the Stellerator model in the mathematical literature by Ildefnso Diaz was a great opportunity to develop new properties for the relative rearrangement. We review some of those properties and we shall give new results related to the relative rearrangement.

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Competition of subspecies and structural stability. Survival of the best adapted or coexistence?

Evariste Sanchez–Palencia *

Equations of population dynamics allow us to consider the competition of two species in an environment. According to the specific properties of their interaction, competition may lead to extinction of one of the species, but also in certain cases to coexistence of both.

In our work, we start from the dynamics of the population of a species; we split it into the populations of two subspecies (of the same specie) with the same properties. It appears that the corresponding dynamical system is structurally unstable, $i.e.$ small perturbations modify drastically the structure of the phase portrait of the dynamics. In that context, we introduce a small perturbation of the properties of one of the subspecies (”mutation”). It clearly appears that, according to the instability of the starting state, the issue of he mutation depends drastically on the very shape of the (even very small) perturbation. Specifically, when the modification may be clearly understood (whatever the circumstances) as an ”advantage” of one of the subspecies on the other, the dynamics leads, generally speaking, to the extinction of the less adapted subspecies. But, *when the modification is ”rather complex”, containing features which are ”advantages” or ”disadvantages” according to the circumstances (in particular initial conditions), the dynamics leads generally to another equilibrium with coexistence of both subspecies*.

Analogous results follow from a starting situation mainly described by two species interacting in a predator / prey framework with a stable periodic cycle. The splitting of one of the species into two subspecies constitutes a structurally unstable system, and the perturbation (mutation) of one of the subspecies may have very different issues. In ”rather complex” situations, the dynamics often leads to one or several periodic cycles with coexistence of the three new species.

On account of the fact that genetic mutations imply modifications *of a specific protein*, which, in its turn has in general an influence on *several behavioral properties*, it clearly appears that *the population dynamics lead to the extinction of the less adapted only in very elementary cases; in usual, more complex cases, it rather leads to the preservation of the variety*.

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Localization properties of solutions of parabolic equations with variable anisotropic nonlinearity

Sergey Shmarev *

We are interested in the character of propagation of disturbances from the data in solutions of parabolic equations with variable and anisotropic nonlinearity. The prototype of such equations is furnished by the equation

$$u_t = \sum_{i=1}^{n} D_i \left(|D_i u|^{p_i(x,t)-2} D_i u \right) + c_0 |u|^{\sigma(x,t)-2} u + f.$$

Anisotropy and variable nonlinearity lead to certain properties intrinsic for the solutions of equations of this type. We prove that unlike the case of isotropic diffusion the solutions vanish in a finite time even in the absence of absorption (i.e. if $c_0 = 0$), provided that the diffusion is fast in only one direction. It is shown that in the case of slow anisotropic diffusion the supports of solutions display a behavior typical for the solutions of equations with strong absorption terms: the support does not expand in the direction corresponding to the slowest diffusion. For certain ranges of the nonlinearity exponents the supports are localized both in space and time. We also discuss the influence of anisotropy on the blow-up of solutions and show that for equations with variable nonlinearity the effects of finite time vanishing and blow-up may happen even if the equation becomes linear as $t \to \infty$. The results were obtained in collaboration with S. Antontsev. The presentation follows the papers [2, 3, 4]; the main tool is the method of local energy estimates developed in [1].

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Travelling waves in a convection-diffusion equation

Peter Takáč *

Abstract: We will discuss existence and stability of travelling waves for a nonlinear convection diffusion equation in the 1-D Euclidean space. The diffusion coefficient depends on the gradient in analogy with the p-Laplacian and may be degenerate or singular. We establish unconditional stability with respect to initial data perturbations in $L^1(\mathbb{R})$. Although our solutions typically do not belong to $L^1(\mathbb{R})$, their difference usually does belong there; therefore, the $L^1(\mathbb{R})$ metric is of crucial importance in our approach.

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Pattern formation: The oscillon equation

Roger Temam *

In this lecture, we will consider the oscillon equation which is used in cosmology to model and represent some transient persistent structures. We will discuss questions of existence and uniqueness of solutions and of long time behavior of solutions

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Nonlinear fractional diffusion equations of porous medium type

Juan Luis Vázquez *

The study of diffusion involving fractional Laplacian operators is a topic of much current research. We present two models for flow in porous media including nonlocal (long-range) diffusion effects of such type.

The first one is based on Darcys law and the pressure is related to the density by an inverse fractional Laplacian operator. We prove existence of solutions that propagate with finite speed, which is unexpected in fractional diffusion models. The model has also the very interesting property that mass preserving self-similar solutions can be found by solving an elliptic obstacle problem with fractional Laplacian for the pair pressure-density. We use entropy methods to show that the asymptotic behaviour is described after renormalization by these solutions which play the role of the Barenblatt profiles of the standard porous medium model.

The second model comes from statistical mechanics considerations, generalizes the well-known linear fractional heat equation, generates a nice nonlinear contractive semigroup, and has infinite speed of propagation for all powers of the nonlinearity.

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POSTER SESSION

Numerical solutions of an elastic-gravitational model by the finite element method

A. Arjona Almodóvar, T. Chacón Rebollo, M. Gómez Marmol, *

Elastic-gravitational model allow the computation of gravity, deformation, and gravitational potential changes in order to investigate crustal deformation Earth (see [2] and [3]). This model can be represented by a coupled system of linear parabolic (for the deformations) and elliptic PD equations (for gravitational potential changes) (see for instance [4], [5] and [1]).

We have considered the internal source as response to the effect of a pressurized magma reservoir into a multilayered, elastic-gravitational earth model.

We present the numerical analysis of such a coupled model by means of the finite element method for the steady model. Finally, we present some numerical tests in meaningful situations confirm the predictions theoretical order of convergence.

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Mathematical analysis of a thermoelastic problem

P. Barral, M.C. Naya-Riveiro and P. Quintela *

In this work we carry out a mathematical analysis of the coupling between the motion and energy conservation equations for thermoelastic materials. Specifically, the existence and uniqueness of a quasistatic problem with mixed displacement-traction and Robin boundary conditions is obtained.

In the literature, there exist several existence results for thermoelastic problems, although not many deal with quasistatic problems with Robin boundary condition. A similar problem was studied by Viaño in [1], and by Figueiredo and Trabucho in [2] considering the dynamic motion equation. We use the same methodology in order to prove the existence of a solution of the problem, although we study the quasistatic problem without contact and for thermoelastic materials, taking into account that our reference temperature depends on the material point, our stress tensor has a particular thermal part and we consider a Robin boundary condition for the temperature. These contributions cause some difficulties to calculate *a priori* estimates. Furthermore, in order to prove the uniqueness their technique was not useful to us and the result is obtained following the methodology of Gawinecki [3].

Let $\Omega \subset \mathbb{R}^n$ (with $n = 2, 3$) be an open and bounded set with smooth boundary and let $[0, t_f]$ be the time interval of interest. Then, the problem we are going to study is finding a displacement field $\mathbf{u}(p, t)$ and a temperature field $\theta(p, t)$ in $\Omega \times (0, t_f]$, verifying:

- the equilibrium equations in $\Omega \times (0, t_f]$

$-\mathrm{Div}\, \boldsymbol{\sigma}(\theta, \mathbf{u}) = \mathbf{b}$	$\boldsymbol{\sigma}(\theta, \mathbf{u}) = \Lambda^{-1} : \boldsymbol{\varepsilon}(\mathbf{u}) - \alpha(\theta - \theta_r)(3\lambda + 2\mu)\mathbf{I}$
	Λ^{-1}: The inverse elasticity tensorial function.
	$\boldsymbol{\varepsilon}(\mathbf{u})$: The deformation tensor.
	α: The coefficient of thermal expansion.
	θ_r, $\mathbf{b}$: The reference temperature and the body forces.
	λ, μ: The Lamé's parameters.
$\rho_0 c_F \dot{\theta} = -\theta_r \alpha(3\lambda + 2\mu)\mathrm{Div}\, \dot{\mathbf{u}} + \mathrm{Div}\,(k\nabla\theta) + f$	ρ_0: The reference density.
	c_F: The specific heat at constant deformation.
	k: The thermal conductivity of the material.
	f: The body heat.

- the mixed displacement-traction conditions and Dirichlet, Neumann and Robin boundary conditions in temperature.

Under appropriate assumptions we prove the existence and uniqueness of solution using the Galerkin's method.

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Nodal and multiple solutions for a prescribed mean curvature problem

Ann Derlet *

We present some existence and multiplicity results for *sign-changing* solutions of the nonlinear mean curvature problem

$$(P)\begin{cases} -\operatorname{div}\big(\nabla u/\sqrt{1+|\nabla u|^2}\big) = \lambda |u|^{p-2}u & \text{in } \Omega,\\ u = 0 & \text{on } \partial\Omega,\end{cases}$$

where $\lambda > 0$ and $2 < p < 2^*$. For instance, if the parameter λ is sufficiently large then (P) possesses at least one nodal solution u_λ with exactly two nodal domains. In addition, it is possible to prove the existence of arbitrarily many nodal solutions of (P), again for large values of λ.

Unlike many authors, we do not look for solutions of bounded variation. Instead we consider a perturbation of the degenerate part $1/\sqrt{1+s^2}$ and work in the Sobolev space $H_0^1(\Omega)$. This allows us to use classical variational techniques such as the Nehari manifold, the nodal Nehari set, and a symmetric version of the mountain pass theorem.

The above results have been obtained in collaboration with D. Bonheure (Brussels) and S. de Valeriola (Louvain-la-neuve).

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The probabilistic Brosamler formula revisited on some nonlinear Neumann boundary problems

Gregorio Díaz *

The principal aim is to study some kind of boundary value problems with nonlinear monotone condition

$$\begin{cases} -\Delta u + \lambda u &= f \qquad \text{in } \Omega \\ \langle \nabla u, \vec{\gamma} \rangle + \rho |u|^{\mathrm{m}-1} u &= \Phi \qquad \text{on } \partial\Omega, \end{cases}$$

for $\lambda \geq 0,\ \rho > 0$ and $\mathrm{m} > 1$, where $\vec{\gamma}$ is an *oblique exterior vector* and $\partial\Omega$ consists only of regular points. Suitable approximations enable us to solve the problem by a *representation formulae* of the viscosity solution as a value function of a single Optimal Control problem controlled by a stochastic and reflecting dynamic system. Although other generalizations are possible, by simplicity we limit our contribution to the presence of nonlinear terms exclusively on the boundary of the domain.

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Numerical Analysis of a closed loop Thermosyphon model with a viscoelastic fluid

Ángela Jiménez-Casas, Mario Castro and Justine Yasappan *

We analyze the motion of a viscoelastic fluid in the interior of a closed loop thermosyphon under the effects of natural convection and a given external heat flux. Numerical experiments are performed in order to describe the behavior of the solution for different ranges of the relevant parameters.

$$\left\{ \begin{array}{l} \varepsilon \frac{d^2 v}{dt^2} + \frac{dv}{dt} + G(v)v = \oint Tf, v(0) = v_0, \frac{dv}{dt}(0) = w_0 \\ \frac{\partial T}{\partial t} + v\frac{\partial T}{dx} = h(x,v,T) + \gamma \frac{\partial^2 T}{\partial x^2}, T(0,x) = T_0(x) \end{array} \right. \tag{1}$$

Where $v(t)$ is the velocity, $T(t,x)$ is the distribution of the temperature of the viscoelastic fluid in the loop, γ is the temperature diffusion coefficient,$G(v)$ is the friction law at the inner wall of the loop, the function f is the geometry of the loop and the distribution of gravitational forces, $h(x)$ is the heat flux and ε is the viscoelastic parameter. Suitable parameters are chosen to carry out the different numerical analysis. The numerical experiments are summarized for a detailed analysis of the behaviour of the system. The experiments made in this poster come to verify the complex nature of the behavior of the models of the thermosyphon system.

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Vortices in a cylindrical annulus

M. C. Navarro and H. Herrero *

Thermal convection has been shown to be determinant in the formation and intensity of some meteorological events such as dust devils and cyclones: dust devils are likely to form in the presence of large horizontal temperature gradients [1], and the evolution of hurricane intensity depends, among other factors, on the heat exchange with the upper layer of the ocean under the core of the hurricane [2, 3].

We consider a fluid in a cylindrical annulus heated from below with a Gaussian profile, the governing equations are the incompressible Boussinesq Navier-Stokes equations. For the numerical implementation, non-linearities are treated with Newtons method. For the discretization (for basic state and linear stability analysis) we use a spectral method by expanding the fields in Chebyshev polynomials and evaluating at the Gauss-Lobatto points [4, 5]. Convergence properties of the numerical method depend on the parameters present in the problem.

Under certain thermal and geometrical conditions, a stable vortex, very similar to a dust devil, can be generated from a convective instability. The horizontal temperature gradient at the bottom of the annulus and the vertical temperature gradient, determine the intensity of the vortex formed and its behavior can be controlled thermally by cooling or heating adequately the bottom boundary [6]. These results connect with that observed for the evolution of the intensity of cyclones and dust devils.

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Modelling a High-Pressure Shift Freezing Process

Stephen Peppin[1], Ángel Manuel Ramos[2] and Nadia Smith[2] *

Freezing is a widespread food preservation technology, as it ensures high food quality with long storage duration, and also because it has an extended implementation area (meat, fish, fruit and vegetables, dairy and egg products, etc.). Despite the benefits, freezing of foods can also cause undesirable changes in their texture and organoleptic properties, and its main drawback is the risk of food damage due to the formation of ice crystals. The size and location of the crystals formed during the freezing process depend on the freezing rate (slow freezing produces large crystals, whilst rapid freezing promotes intensive nucleation and the formation of small ice crystals) and the final temperature of the process. The general purpose of food technologists working on this area has been to create a homogeneous matrix of small ice crystals. Improvement of known freezing methods and development of new techniques are important research goals for the food industry at present. With the recent increasing impact of High-Pressure technology on Food Processing, there has been a lot of research dealing with the potential applications of High-Pressure effects on ice-water transitions, given that pressure decreases the freezing and melting point of water to a minimum of $-22°$C at 207.5 MPa, namely High-Pressure Freezing and Thawing.

One particular case of High-Pressure Freezing is High-Pressure Shift Freezing (HPSF), in which phase transition occurs due to a pressure change that promotes metastable conditions and instantaneous ice production. On expansion, pressure release occurs instantaneously throughout the product (Pascal principle), and subsequently, its temperature decreases. Large-scale supercooling takes place throughout the sample, which implies high ice nucleation velocities. Different authors have proved experimentally that ice nucleation occurs homogeneously throughout the whole volume of the product and not only on the surface, as they have found small granular shape ice crystals disperse throughout the resulting sample for several products. When comparing HPSF to classical freezing processes, important reductions of freezing times have been reported.

In this work we derive a model for a HPSF process of a solid type food, with a big and small filling sample vs pressurizing media ratio. We present a heat transfer model derived from an enthalpy formulation based on volume fractions dependent on temperature and time, that simulates the temperature profile during a HPSF process, calculating also the amount of ice instantaneously formed after expansion and the supercooling of the sample.

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On the Lipschitz Property of the Relative Rearrangement

María Luisa Seoane *

Since its introduction as the derivative of the decreasing rearrangement u_*, several properties of the relative rearrangement b_{*u} have been established, in particular the Lipschitz condition considering the relative rearrangement like a function of b. Here, our interest will be focused on the relative rearrangement considered as a function of u which is a new feature as far as we know.

We give some useful estimations for the numerical solution of differential equations involving relative rearrangements of a data function b with respect to the solution u, when fixed point techniques are used. This kind of nonlocal problems is widely discussed in the literature dealing the plasma physics. In order to prove the convergence of iteration schemes it is helpful to have a Lipschitz condition (even more, a contractive property) of nonlocal terms. We shall just suggest a way to obtain them and analyse some suitable hypothesis.

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Mathematical treatment of a climatological model with deep ocean effect

Lourdes Tello *

This poster is a review of some results obtained in collaboration with J.I. Díaz in the last years. We have studied the existence and multiplicity of solutions to several transient and equilibria simple climate models.

In the last decades, many authors have studied the so-called global climate energy balance models (EBM) dealing with the evolution of the mean surface temperature of the Earth. Among them we can mention D. Arcoya, R. Bermejo, J.I. Díaz, M. Ghil and S. Childress, J. Hernández, G. Hetzer, G.R. North, B. Schmidt, X. Xu, etc.

In many of the previous models, the effect of the oceans is only considered in an implicit and empirical way in the spatial dependence of the coefficients. However, some works about the rapid climatic change in Glacial-Holocene transition (see Berger et al) show that it could be related to the past changes in deep water formation. In this work we study a model including the effect of the deep ocean based on the model proposed by Watts - Morantine.

The simplified model represents the evolution of the temperature U in a global ocean Ω with constant depth H. The upper boundary of Ω simulates the Earth surface. The governing equation for the ocean interior is parabolic and the upper boundary condition comes from an energy balance for the mean surface temperature of the Earth. In such energy balance, the absorbed energy depends on the planetary coalbedo β (which is discontinuous on U). The coalbedo is treated in the general class of multivalued graphs. More precisely, we shall assume that β is a bounded maximal monotone graph of $\mathbb{R}^2$. Other nonlinearity at the boundary concerns the surface diffusion proposed by P.H. Stone: its diffusion coefficient depends on the temperature gradient in order to include the negative feedback of the eddy fluxes. So, the energy balance involves in this way the p-Laplacian surface operator.

This kind of models is very sensitive to small fluctuations of Solar and terrestrial parameters. We also analyze how the Solar constant is related to the number of equilibrium solutions. One of the main difficulties in the mathematical treatment of these models comes from the presence of a nonlinear dynamic and diffusive boundary condition in its formulation.

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MORE CONTRIBUTIONS TO APPEAR IN THE SPECIAL ISSUE OF DIFFERENTIAL EQUATIONS AND APPLICATIONS

A variational approach to camera motion smoothing

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In this paper we deal with camera motion smoothing. We focus our attention on the case of video cameras mounted on a tripod. In such case, for each time t of the video sequence, camera motion configuration is provided by 3 parameters : (1) $P(t)$ (PAN) which represents the tripod vertical axis rotation, (2) $T(t)$ (TILT) which represents the tripod horizontal axis rotation and (3) $Z(t)$ (CAMERA ZOOM) the camera lens zoom setting. From these 3 parameter functions we can easily deduce (using tripod information), for each time t, the camera calibration, that is, the camera intrinsic and extrinsic parameters which determine the position of the camera in the $3D$ space and the way $3D$ objects are projected in the camera projection plane (the CCD in the case of digital cameras).

Human being visual system is very sensitive to motion, and small perturbations over time of $P(t)$, $T(t)$, and $Z(t)$ values produce small oscillations in the camera motion disturbing the observer. To remove such perturbations is a critical issue in applications like the inclusion of artificial graphic objects in real video sequence scenarios. To illustrate this phenomenon, in http://www.ctim.es/demo102 we show a real video sequence where some graphic objects have been included. In this video we illustrate the main practical problem we deal with, that is, standard calibration techniques which do not take into account the expected time regularity of $(P(t), T(t), Z(t))$ can introduce a significant noise in the camera motion estimation over time.

In this paper we propose to smooth $(P(t), T(t), Z(t))$ functions by minimizing the following energy :

$$E(P(t), T(t), Z(t)) = \int_{t_0}^{t_1} \left(w_P P'(t)^2 + w_T T'(t)^2 + w_Z Z'(t)^2 + F(P(t), T(t), Z(t), t) \right) dt, \qquad (1)$$

where $[t_0, t_1]$ is the time interval, $w_P, w_T, w_Z \geq 0$ are weights to balance the different components of the energy and $F(x_1, x_2, x_3, t) \geq 0$ is an standard calibration function which forces, for each time t, that the projection of $3D$ points be close to primitives detected in the image. In fact, when no time regularization is used, $(P(t), T(t), Z(t))$ is usually estimated by minimizing $F(P(t), T(t), Z(t), t)$ independently for each time t. So, by using the proposed variational model (1), we introduce a time regularity condition in the video camera calibration procedure. In http://www.ctim.es/demo102 we illustrate the results we obtain by applying the proposed method. We include the smoothed video sequence and some graphics to compare original and smoothed $(P(t), T(t), Z(t))$ functions

The Euler-Lagrange equations associated to energy (1) yields to the following nonlinear system of differential equations:

$$\begin{cases} -w_P P''(t) + \frac{\partial F}{\partial x_1}(P(t), T(t), Z(t), t) = 0 \quad in \quad (t_0, t_1) \\ -w_T T''(t) + \frac{\partial F}{\partial x_2}(P(t), T(t), Z(t), t) = 0 \quad in \quad (t_0, t_1) \\ -w_Z Z''(t) + \frac{\partial F}{\partial x_3}(P(t), T(t), Z(t), t) = 0 \quad in \quad (t_0, t_1), \end{cases}$$

with adequate boundary conditions. In this paper we study the minimization problem (1) and we present some numerical experiments to illustrate the smoothing performance of the proposed method.

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An inverse problem related to temperature distribution on steel under fire conditions

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The temperature distribution inside a steel plate within a hight temperature field follows a non linear heat equation where conductivity and thermal capacity depends on absolute temperature. The faces of the plate are exposed to an oxidation process that produces two time-dependent narrow regions where conductivity decreases strongly. Consequently, there are two free boundaries separating the inner steel from the oxidized one. It has been designed an electrical furnace, which allows to simulate the behaviour of the fire in buildings. It has been registered temperature data in several points of the plate and several instants. We consider the inverse problem of to determine the conductivity so that the model fits the experimental data.

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Estimation of the dynamics of protein folding processes from free energy computations

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The passage of proteins from a denatured state to a biologically functional (native) state is often modeled as a diffusion process along a reaction coordinate[5]. The usual approach of modeling proteins as mechanical systems and conducting Monte Carlo simulations to sample their phase space yields spatial conformations and free energy profiles[3] but not the phase space trajectories corresponding to the folding process. We present an approach to reconstruct the dynamics from the free energy profiles resulting from Monte Carlo simulations. This approach consists in choosing a suitable diffusivity profile and numerically solving the one-dimensional Smoluchowski equation[4]

$$\begin{cases} \dfrac{\partial f}{\partial t}(x,t) = \dfrac{\partial}{\partial x}\left(D(x)\,p(x)\,\dfrac{\partial}{\partial x}\left(\dfrac{f(x,t)}{p(x)}\right)\right), & (x,t) \in [a,b] \times \mathbb{R}_{>0}, \\ f(x,0) = f_0(x), & x \in [a,b], \\ \dfrac{\partial f}{\partial x}(x,t) = \dfrac{p'(x)}{p(x)} f(x,t), & x \in \{a,b\}. \end{cases} \tag{1}$$

where $f(x,t)$ is the probability density function of the distribution of states with reaction coordinate x at time t, $D(x)$ is the diffusivity profile, $p(x)$ is the (temperature-dependent) equilibrium probability, and $f_0(x)$ is the initial probability. From the solution of this equation for a wide array of temperatures, it is possible to extract folding rates that can be matched against experimentally observed rates. We have used global optimization techniques[2] to find the diffusivity profiles (initially assuming $D(x) =$ constant) that, when substituted in (1), yield folding rates that are closest to those obtained in laboratory experiments. Also, our numerical scheme for solving (1) is significantly faster than the most popular one appearing in the Chemical Physics literature[1] while being of the same order of accuracy.

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On the long-term solution of a quasi-geostrophic ocean circulation model: the continuous and the semi-discrete models

Rodolfo Bermejo and Pedro Galán del Sastre *

We study in this paper the long-term solutions of a barotropic quasi-geostrophic ocean circulation model and its semi-discrete finite element formulation. We prove the existence of global attractors for the continuous and the finite element semi-discrete models and the convergence of the semi-discrete attractor to the continuous one when the space discretization parameter $h \to 0$.

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A variance-expected compliance approach for topology optimization

Miguel Carrasco [*], **Benjamin Ivorra** [†], **Rodrigo Lecaros** [‡], **Angel M. Ramos** [†]

We consider an elastic homogeneous body $\Omega \subseteq \mathbb{R}^d$. We impose some support conditions in $\Gamma_u \subseteq \partial\Omega$,. We apply some external load forces to this body. Our purpose is to find the optimal distribution of material in Ω when the external load force has a stochastic behavior [1].

Assuming the linear response of the body material, the displacements is obtained by solving:

$$-\operatorname{div}(K\,e(u)) = f \text{ in } \Omega; u = 0 \text{ on } \Gamma_u; (K\,e(u)) \cdot n = 0 \text{ on } \partial\Omega \setminus \Gamma_u \cup \Gamma_t; (K\,e(u)) \cdot n = g \text{ in } \Gamma_t, \quad (1)$$

where $\Gamma_u \cap \Gamma_t = \varnothing$, f corresponds to an external load, g is a surface force applied to Γ_t, $u\colon \Omega \to \mathbb{R}^d$ is the vector of displacements, $e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ denotes the strain tensor and $K = (K_{i,j,k,l})$ is the elasticity tensor. Under suitable conditions, the solution of (1) is unique [2].

We assume that K depends on a parameter $\lambda \in \Lambda$ measuring the amount of material at each point of Ω, Λ is the set of admissible material distribution, and $g \equiv 0$. The minimum compliance design problem can be stated as following [2]:

$$\min\{l(u(\lambda)) \mid \lambda \in \Lambda; A[\lambda](u(\lambda), v) = l(v), \text{for all } v \in \hat{H}\}, \quad (2)$$

where $A[\lambda](u,v) = \int_\Omega K_{i,j,k,l} e_{i,j}(u) e_{i,j}(v)\,\mathrm{d}x$, $l(u) = \int_\Omega f \cdot u\,\mathrm{d}x$, $\hat{H} = \{u \in [H^1(\Omega)]^d \mid u_i|_{\Gamma_u} = 0,\, i = 1, \ldots, d\}$ and $u(\lambda)$ denotes the unique weak solution of (1).

We consider that f is randomly perturbed by the random vector ξ (with $\mathbb{E}(\xi) = 0$). Thus, a stochastic topology design problem can be stated as [3]

$$\min\{\alpha \mathbb{E}[\Psi(\xi,\lambda)] + \beta \operatorname{Var}[\Psi(\xi,\lambda)] \mid \lambda \in \Lambda\}, \quad (3)$$

where $\alpha, \beta \geq 0$, $\alpha + \beta > 0$, $\mathbb{E}(\cdot)$ and $\operatorname{Var}(\cdot)$ are the expected value and the variance of the corresponding random function, respectively, and $\Psi(.,.)$ is defined by

$$\Psi(\xi,\lambda) = \quad \{\textstyle\int_\Omega (f+\xi) \cdot u\,\mathrm{d}x \mid u \in \hat{H}, \text{ s.t.: } A[\lambda](u,v) = \int_\Omega (f+\xi) \cdot v\,\mathrm{d}x, \text{ for all } v \text{ in } \hat{H}\}. \quad (4)$$

In this work, we use previous results obtained for truss structures [1, 3] to give an explicit expression of (3) and solve it by using a global optimization approach [4]. Then, we show that a good choice of (α, β) allows to obtain a material distribution of the body Ω stable to the considered load perturbations.

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Finite extinction and control in some delay models

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For a controllable linear time-invariant system a general delayed feedback action is proposed so that the solutions of the corresponding closed-loop system are driven to zero in finite time.

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Physical viscosity numerical methods for non-conservative hyperbolic systems

M.J. Castro, M.L. Muñoz, C. Parés *

The design of numerical methods for nonconservative hyperbolic systems of the form

$$w_t + A(w)w_x = 0$$

is a very active front of research, as PDE systems of this nature arises in many flow models. One of the main difficulties both from the theoretical and the numerical point of view is that, for discontinuous solutions, the nonconservative products $A(w)w_x$ cannot be defined within the framework of the distributions. Nevertheless, there are different mathematical theories that allow one to define these products as Borel measures: sea [1]. Unfortunately, these theories lead to different notion of weak solutions: in particular, the jump conditions across a discontinuity depend on the chosen notion. Therefore, an important issue is to choose the definition of weak solutions in such a way that the jump conditions are in good agreement with the physics of the problem.

When the hyperbolic system is the vanishing-viscosity limit of the parabolic problems

$$w_t^\epsilon + \mathcal{A}(w^\epsilon)\, w_x^\epsilon = \epsilon(\mathcal{R}(w^\epsilon) w_x^\epsilon)_x, \tag{1}$$

where $\mathcal{R}(w)$ is a positive semi-definite matrix, the physical jump conditions should be related to the viscous profiles of the regularized problem (1). While in the case of conservative systems the usual Rankine-Hugoniot conditions are recovered independently of the choice of $\mathcal{R}(w)$, this is not the case for non-conservative systems. Therefore, the definition of weak solutions should be such that the corresponding jump conditions are consistent with the viscous profiles related to the physical viscosity.

Once the definition of weak solution has been chosen, a second difficulty is related to the design of numerical methods providing approximations whose limits are weak solutions of the system: see [2]. The difficulty comes from the fact that most of the usual discretization methods for P.D.E. involve some numerical diffusion and, due to this, the numerical approximations converge to functions that are classical solutions where they are smooth but whose discontinuities are consistent with the numerical viscosity but not with the physical one. This phenomenon has been first studied by T. Hou and P. LeFloch [3] when a non-conservative numerical scheme is applied to a system of conservation laws.

The strategy proposed here is to consider numerical schemes which are obtained by adding a physical viscosity term to a second order Lax-Wendroff method. The numerical viscosity coincides thus with the physical one so that the convergence to the correct weak solutions is expected to be improved. This improvement will be illustrated by applying this strategy to he strategy to a non-conservative coupled Burgers system.

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Eigenvalue Problems With Fully Discontinuous Operators

Houssam Chrayteh[1]

We introduce the notion of $\overrightarrow{\rho}$–multivoque Leray-Lions operator

$$Au = -\operatorname{div}_{\overrightarrow{\rho}}\left(\partial\Phi_i\left(x, \frac{\partial u}{\partial x_i}\right)\right), \qquad \overrightarrow{\rho} = (\rho_0, ..., \rho_N)$$

that are strongly monotonic on a Banach-Sobolev function space V and we study the generalized eigenvalue problem $Au = \lambda\partial_j(u)$. Here $\partial\Phi_i$ and ∂_j denote the subdifferential in the sense of convex analysis or more generally in the sense of H. Clarke.

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Numerical identification of a time-dependent conductivity coefficient

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This work deals with the approximation of the solution of the inverse problem of determining the conductivity coefficient, when it depends on time. We asume that temperature is known at some points of the medium and these values can be affected by measurement errors. Such a situation arises, for example, in the context of food technology.

We consider a heat transfer equation with a source term depending on temperature and pressure increase. This equation is completed with appropriate initial and boundary conditions. More precisely,

$$\begin{cases} \varrho C \dfrac{\partial T}{\partial t} - k(t)\Delta T = \alpha p'(t) T & \text{in } \Omega \times (0, t_{\mathrm{f}}) \\ k(t)\dfrac{\partial T}{\partial \vec{n}} = h\left(T_{\mathrm{ref}}(t) - T\right) & \text{on } \partial\Omega \times (0, t_{\mathrm{f}}), \\ T = T_0 & \text{in } \Omega \times \{0\}. \end{cases}$$

The goal is to solve the inverse problem of determining an approximation of function $k(t)$, assumed some temperature values at the boundary and inside the medium are known and they are affected by the errors caused by the measurement devices.

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Interactions between segregation and spatial adaptation in a population dynamics problem

Gonzalo Galiano *

In *Spatial segregation of interacting species*, J. Theor. Biol. (1979), Sighesada, Kawasaki and Teramoto introduced a system of partial differential equations modeling the evolution of spatial spread and segregation of different but similar populations. Apart from competitive Lotka-Volterra (reaction) and population pressure (cross-diffusion) terms, a convective term modeling the populations attraction to more favorable environmental regions was included. In their work, the convective term was assumed to be linear and determined by a given environmental potential. In this article we introduce a nonlinear and non-local dependence in the convective term which allows us to model spatial adaptation by means of a memory mechanism which strengthen the attraction of a population to a point if the population density in such point has been high in the past. In our contribution, after describing the mathematical problem we briefly discuss its well-possedness and propose a numerical discretization in terms of a mass-preserving time semi-implicit finite differences scheme. We also provide the results of biologically inspired numerical experiments showing qualitative differences between the original model of Sighesada et al. and the model proposed in this article.

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On an induction-conduction PDE system in harmonic regime

María Teresa González Montesinos*   Francisco Ortegón Gallego†

We analyze the mathematical modeling of the heating industrial process by induction-conduction of a steel workpiece. Consider the setting depicted in Figure 1. Here, Ω^{s} stands for the steel workpiece to be hardened, Ω^{c} is the inductor (which is made of copper) and D is an open set containing $\bar{\Omega}^{\mathrm{s}} \cup \bar{\Omega}^{\mathrm{c}}$. We assume that Ω^{s}, Ω^{c} and D are Lipschitz-continuous open sets in $\mathbb{R}^3$, $\Omega^{\mathrm{s}} \cap \Omega^{\mathrm{c}} = \varnothing$, $\bar{\Omega}^{\mathrm{s}} \cap \bar{\Omega}^{\mathrm{c}} = S$. Also the auxiliary cross-section $\Gamma \subset \Omega^{\mathrm{c}}$ is assumed to be a Lipschitz-continuous surface.

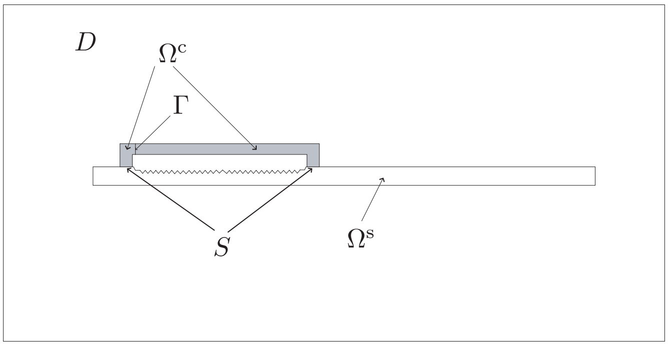

Figure 1: Open sets D, Ω^{s} and Ω^{c}, contact surfaces S, and the auxiliary cross-section Γ.

Neglecting mechanical effects, the mathematical modeling describing this situation is given by the following set of PDEs:

$$\nabla \cdot (\sigma(\theta)\nabla\varphi) = 0 \text{ in } \Omega \times (0,T), \tag{1}$$

$$\frac{\partial\varphi}{\partial n} = 0 \text{ on } \partial\Omega \times (0,T), \tag{2}$$

$$\left[\sigma(\theta)\frac{\partial\varphi}{\partial\nu}\right]_{\Gamma} = \boldsymbol{j} \text{ on } \Gamma \times (0,T), \tag{3}$$

$$i\omega\sigma_0(\theta)\boldsymbol{A} + \nabla \times \left(\frac{1}{\mu}\nabla \times \boldsymbol{A}\right) - \delta\nabla(\nabla \cdot \boldsymbol{A}) + \sigma_0(\theta)\nabla\varphi = 0 \text{ in } D \times (0,T), \tag{4}$$

$$\boldsymbol{A} = \boldsymbol{0} \text{ on } \partial D \times (0,T), \tag{5}$$

$$\rho c_\epsilon \theta_{,t} - \nabla \cdot (\kappa(\theta)\nabla\theta) = \frac{\sigma(\theta)}{2}|i\omega\boldsymbol{A} + \nabla\varphi|^2 + G \text{ in } \Omega \times (0,T), \tag{6}$$

$$\frac{\partial\theta}{\partial n} = 0 \text{ on } \partial\Omega \times (0,T), \tag{7}$$

$$\theta(\cdot,0) = \theta_0 \text{ in } \Omega. \tag{8}$$

We have studied the system (1)-(8) and established some existence results under certain assumptions on data.

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Numerical analysis of a climate model

Arturo Hidalgo [*] **and Lourdes Tello** [†]

In this work we consider a coupled model surface/deep ocean, which was first proposed by Watts-Morantine (1990). It is a diagnostic model which can be used to understand the long-term climate evolution. The unknown is the temperature over each parallel and the effect of the deep ocean on the temperature of the Earth surface is considered. One of the main difficulties of this problem is the dynamic and diffusive boundary condition. The purpose of this work is to develop a numerical scheme to obtain an approximate solution of the coupled model. The numerical technique used is based on the finite volume method together with WENO reconstruction and a Runge-Kutta TVD scheme for time discretization. As an important consequence, we analyze the behaviour of the solution of the energy balance model with and without the effect of deep ocean. This sort of climate models has been extensively studied by J.I. Díaz.

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Metastable Solutions for the Thin-Interface Limit of a p-Laplacian Phase Field Model

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We consider a generalization of the semilinear phase field model from [1, 4, 5], using a non-linear diffusion operator (p-Laplacian) for the phase field function; in particular, the following one-dimensional nonlinear parabolic system

$$\left\{ \begin{array}{rcll} \tau\varphi_t & = & \xi^p(|\varphi_x|^{p-2}\varphi_x)_x - g'(\varphi) + ch'(\varphi)u, & x \in (a,b) \\ u_t & = & ku_{xx} - \frac{l}{2}(h(\varphi))_t, & x \in (a,b) \\ \varphi'(a) & = & \varphi'(b) = 0 & \\ u'(a) & = & u'(b) = 0 & \\ \varphi(0,x) & = & \varphi_0(x) \in H^1(a,b) & \\ u(0,x) & = & u_0(x) \in L^2(a,b) & \end{array} \right. \quad (1)$$

Here $u(t,x)$ represents the temperature of the point x at time t of a substance which may appear in two different phases, (for example liquid-solid) and $\varphi(t,x)$ is the phase field function or order parameter, which represents the local phase average. The functions $g(\varphi)$ and $h(\varphi)$ are positive , such that $g'(\varphi)$ is typically $\frac{1}{2}(\varphi^3 - \varphi)$, and $h'(\varphi) = (1-\varphi^2)^a$ with $a = 0, 1$ (see[6]).The positive constants l and k refer to latent heat and diffusivity, whereas ξ (interface width) and τ are positive parameters related to time and length scales. Finally c is also a positive constant which measures the strenth of the coupling between the fields (see [1]) and the enthalpy function given by $v = u + \frac{l}{2}h(\varphi)$, allows to study more general couplings between a diffusion field and a phase-field. For instance, the phase-field can be seen as the density of bacterial collony or the mass of growing tumor. Analogously, the diffusion field can stand for the density of nutrient.

We study the behavior of the solutions with $\tau = \tau(\xi)$, when ξ goes to zero (thin-interface limit [6]). The main objective of this work is to prove the existence of the metastable solutions of the generalized system (1),($p \neq 2$) that evolve very slowly in time (see [2, 3, 4, 5].)

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Nonlinear models in partial differential equations arising in nuclear fusion

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We carry out mathematical analysis of some bidimensional nonlinear problems satisfied by the averaged poloidal flux of the magnetic field in the magnetic confinement of a plasma in the nuclear fusion. We show some mathematical models related to the stationary and evolution regime of a plasma in Tokamak and Stellarotor devices. The models can be formulated as an inverse problems since several nonlinear terms of the partial differential equation are not a priori known (non local terms). Using the current balance within each flux magnetic and the notion of relative rearrangement we can reformulate the problem as a non local one but having a direct formulation. We review some models concerning to Stellarator devices and we review some results about existence, uniqueness and regularity of solutions.

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Internal degrees of freedom in perturbed nonlinear Klein-Gordon equations

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Abstract

We investigate the kink solutions to the generalized nonlinear Klein-Gordon equation in the presence of inhomogeneous forces and nonlocal operators.

We have found that the number of kink internal modes can depend on the asymptotic behavior of the kink solution for large values of $|x|$.

A list of mechanisms that are capable to create new kink internal modes would contain some of the following items: inhomogeneous perturbations that generate unstable equilibrium positions for the kink, extended de-localized and space-dependent perturbations, external perturbations that do not decay exponentially, and nonlocal operators.

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